

EXPERIMENTAL INVESTIGATION ON FLOW
STRUCTURE AND DISCHARGE DISTRIBUTION IN
T-SHAPE OPEN CHANNEL BIFURCATIONS OF
VARIABLE WIDTHS

BY

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ABSTRACT

In this research, open-channel bifurcations, defined as geometrical singularities of a hydrographic network, where the inflow separates into two downstream branches are investigated. Bifurcations are naturally present in river networks, mostly upstream from islands and in deltas, where the main flow does not necessarily continue in a straight direction but may form a dimensionally asymmetrical Y-shaped diverging channel. Sediment transport in bifurcations are increasingly becoming a major issue when extensive suspended as well as bed material conveyed by rivers regularly encroach into side or lateral (i.e. bifurcated) channels utilized for abstracting raw water thus affecting optimal intake for potable consumption. These divisions in flow are also observed in human-intervened conditions such as wastewater networks where the combined sewer overflows evacuate part of the exceeding sewer discharge capacity toward the surrounding environment. In addition, flooded streets reaching three-branch crossroads generate bifurcated channel flows. For most design or man-made cases, the main flow continues straight ahead, while the side branch, generally narrower, makes an angle close to 90° with the main channel, forming a junction known as T-shape bifurcation. Many researchers have completed studies on T-shape bifurcations to understand the behaviour of water flow and sediment transport. However, a complete understanding of the phenomenon, especially in relation to 2D or 3D secondary flows and vortices, is lacking up to this day. Moreover, the distribution of flow discharges in both main and lateral/branch channels requires further detailed investigation. As such, the objectives of this research on open-channel flow in 90° bifurcations are to determine experimental techniques of flow visualization, investigate methods to identify three-dimensional flow patterns, improve empirical discharge distribution relationships, and establish the effect of reduced side branch width on flow distribution. A total of 668 experimental runs have been conducted and completed in the open-channel flume system at Laboratoire de Mécanique des Fluides et d'Acoustique (LMFA) of Institut National des Sciences Appliquées (INSA) in Lyon, France. From the results, it is observed that flow visualization techniques were generally not effective nor adequately consistent to differentiate between 2D standard recirculation or 3D helical flow structures. Therefore, another approach based on analysis of fluid power gain or loss plotted against inflow discharge in the main channel is introduced as an alternative. As for assessing flow division through the bifurcation, the results are organized into sections to progressively address discharge distributions in the subcritical or free recirculation transcritical regimes. Consequently, a new predictive set of empirical equations for flow distribution is derived based on the known upstream discharge plus branch-width ratio and stage-discharge relationships in the downstream or lateral/branch channels. This discharge distribution formulation is accurate, as indicated by determination coefficients of at least 0.98 when compared to measurements, without even identifying beforehand whether the flow is in fully subcritical or free recirculation transcritical regime through the lateral/branch channel. Based on the successful accomplishment of stipulated objectives, recommendations for further work to include establishment of a flow-structure identification system, as well as extending the experiments to bifurcations with significantly narrower side branches. The angle of flow divergence could also be varied to other than 90° .

خلاصة البحث

في هذا البحث، تم دراسة التشعبات مفتوحة القناة، والتي تم تعريفها على أنها تفردات هندسية لشبكة هيدروغرافية، حيث ينفصل التدفق إلى فرعين في اتجاه مجرى النهر. توجد التشعبات بشكل طبيعي في شبكات الأنهار، ومعظمها عند المنبع من الجزر وفي الدلتا، حيث لا يستمر التدفق الرئيسي بالضرورة في اتجاه مستقيم ولكنه قد يشكل قناة متباعدة غير متماثلة الأبعاد على شكل حرف Y. أصبح نقل الرواسب في التشعبات مشكلة رئيسية بشكل متزايد عندما تتعدى المواد المعلقة وكذلك المواد القاعية التي تنقلها الأنهار بانتظام إلى القنوات الجانبية أو الجانبية (أي المتشعبة) المستخدمة لاستخراج المياه الخام مما يؤثر على المدخول الأمثل للاستهلاك الصالح للشرب. هذه الانقسامات في التدفق لوحظت أيضًا في الظروف التي يتدخل فيها الإنسان مثل شبكات مياه الصرف الصحي حيث تعمل فيضانات المجاري المجمعة على تفريغ جزء من قدرة تصريف المجاري الزائدة نحو البيئة المحيطة. بالإضافة إلى ذلك، فإن الشوارع التي غمرتها المياه والتي تصل إلى تقاطعات الطرق الثلاثة تولد تدفقات قناة متشعبة. بالنسبة لمعظم حالات التصميم أو تلك التي من صنع الإنسان، يستمر التدفق الرئيسي للأمام مباشرة، في حين أن الفرع الجانبي، وهو أضيق عمومًا، يصنع زاوية قريبة من 90 درجة مع القناة الرئيسية، ويشكل تقاطعًا يعرف باسم التشعب على شكل حرف T. أكمل العديد من الباحثين دراسات حول التشعبات على شكل حرف T لفهم سلوك تدفق المياه ونقل الرواسب. ومع ذلك، لا يتوفر فهما كاملا لهذه الظاهرة، خاصة فيما يتعلق بالتدفقات والدوامات الثانوية ثنائية أو ثلاثية الأبعاد، حتى يومنا هذا. علاوة على ذلك، فإن توزيع تصريفات التدفق في كل من القنوات الرئيسية والجانبية/الفرعية يتطلب مزيدًا من التحقيق التفصيلي. وعليه، فإن أهداف إجراء هذا البحث حول تدفق القناة المفتوحة في التشعبات بزوايا 90 درجة هي هي تحديد التقنيات التجريبية لتصوير التدفق، والبحث في طرق تحديد أنماط التدفق ثلاثية الأبعاد، وتحسين علاقات توزيع التفريغ التجريبية، وتحديد تأثير انخفاض عرض الفرع الجانبي على توزيع التدفق. تم إجراء وإكمال ما مجموعه 668 عملية تجريبية في نظام المسائل ذي القناة المفتوحة في مختبر ميكانيكا السوائل والصوت (LMFA) التابع للمعهد الوطني للعلوم التطبيقية (INSA) في ليون، فرنسا. من النتائج، لوحظ أن تقنيات تصور التدفق لم تكن فعالة بشكل عام ولا متسقة بشكل كافٍ للتمييز بين إعادة التدوير القياسية ثنائية الأبعاد أو هياكل التدفق الحلزونية ثلاثية الأبعاد، لذلك، تم تقديم نهج آخر يعتمد على تحليل اكتساب أو فقدان طاقة الموائع كبديل. أما بالنسبة لتقييم تقسيم التدفق من خلال التشعب، فقد تم تنظيم النتائج في أقسام لمعالجة توزيعات التصريف تدريجياً في أنظمة إعادة التدوير دون الحرجة أو الحرة عبر الحرجة. وبناءً على ذلك، تم اشتقاق مجموعة تنبؤية جديدة من المعادلات التجريبية لتوزيع التدفق استناداً إلى تفريغ المنبع المعروف بالإضافة إلى نسبة الفرع إلى العرض وعلاقات المرحلة-التفريغ في القنوات النهائية أو الجانبية/الفرعية. صيغة توزيع التفريغ هذه دقيقة، كما يتضح من معاملات التحديد التي لا تقل عن 0.98 عند مقارنتها بالقياسات، دون حتى تحديد مسبق لما إذا كان التدفق يتواجد في نمط دون الحرج بشكل كلي أو نمط إعادة تدوير عبر الحرج خلال القناة الجانبية / الفرعية. بناءً على الإنجاز الناجح للأهداف المحددة، يوصى بمواصلة العمل من خلال إنشاء نظام تحديد مبني على التدفق - البنية، وكذلك توسيع التجارب لتشمل التفرعات ذات الفروع الجانبية الأضيق بقدر مؤثر. يمكن أيضاً تغيير زاوية انحراف التدفق عن 90 درجة.

APPROVAL PAGE

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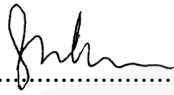
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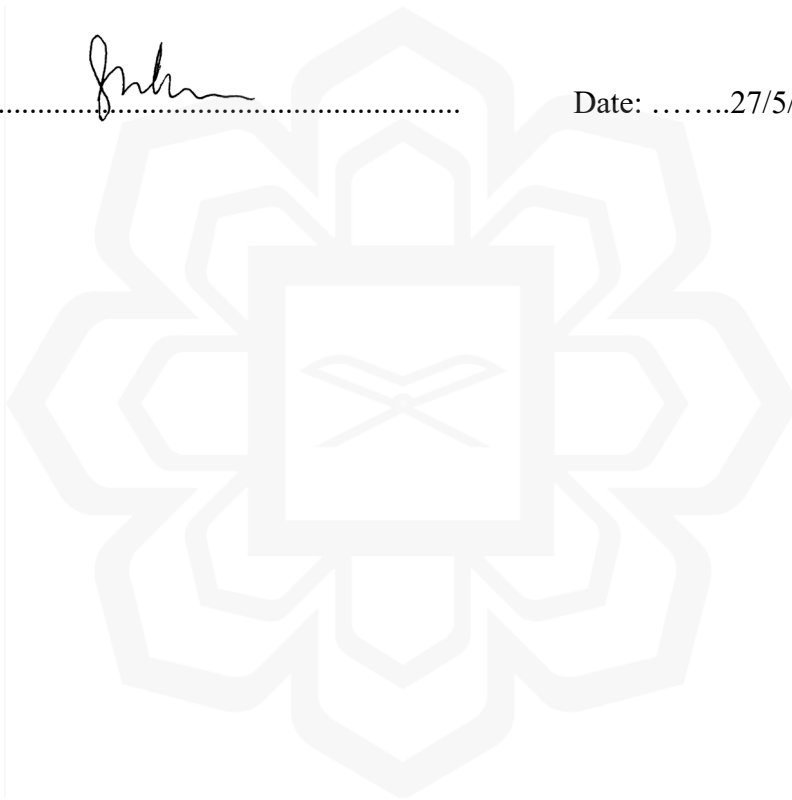
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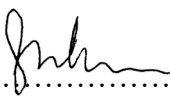
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In the Name of Allah, the Most Compassionate, the Most Merciful

With the Blessings of Allah, the Most Merciful, the Most Compassionate, I begin this acknowledgement. All Glory and Praise belongs to Allah, the Sovereign Lord of all creation, and, peace be upon our beloved Prophet Mohammed (peace be upon Him).

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LIST OF SYMBOLS

B	width of sharp-crested weir (Rehbock, 1929)
b_x	main channel width
b_y	branch channel width
C_x	downstream weir height in main channel (Momplot et al, 2017)
C_y	downstream weir height in lateral/branch channel (Momplot et al, 2017)
F_{bx}	upstream Froude number in main channel
F_{Cx}	Froude number based on main channel downstream weir height (Rivière et al, 2007)
ρ	density of water
F_{xo}	downstream Froude number in main channel
F_{yo}	Froude number in lateral/branch channel
g	gravitational acceleration
h	flow depth above weir (Rehbock, 1929)
h_d	downstream flow depth in main channel (Momplot et al., 2017)
h_l	flow depth in lateral/branch channel (Momplot et al., 2017)
h_u	upstream flow depth in main channel (Momplot et al., 2017)
h_x	upstream flow depth in main channel
h_{xo}	downstream flow depth in main channel
h_{yo}	flow depth in lateral/branch channel
$h_{crit,x}$	critical depth in upstream main channel
$h_{crit,xo}$	critical depth in downstream main channel
$h_{crit,yo}$	critical depth in lateral/branch channel
μ	dynamic viscosity of water
k_s	equivalent roughness elevation
P	height of weir (Rehbock, 1929)

Q_d	downstream discharge or flowrate in main channel (Momplot et al., 2017)
Q_l	discharge or flowrate in lateral/branch channel (Momplot et al., 2017)
Q_u	upstream discharge or flowrate in main channel (Momplot et al., 2017)
Q_{xi}	upstream discharge or flowrate in main channel
Q_{xo}	downstream discharge or flowrate in main channel
Q_{yo}	discharge or flowrate in lateral/branch channel
r^2	coefficient of determination
R_b	ratio of channel widths, i.e. lateral/branch to main ($\frac{b_y}{b_x}$)
R_c	ratio of weir heights [Rivière et al (2007)]
Re_{bx}	upstream Reynolds number
R_q	discharge distribution or ratio ($\frac{Q_{yo}}{Q_{xi}}$)
R_{q-sub}	discharge distribution ($\frac{Q_{yo}}{Q_{xi}}$) for unequal weir heights ($C_x \neq C_y$) [Rivière et al (2007)]
R_{q-sym}	discharge distribution ($\frac{Q_{yo}}{Q_{xi}}$) for equal weir heights ($C_x = C_y$) [Rivière et al (2007)]
σ	air/water surface tension
θ	bifurcation angle

LIST OF ABBREVIATIONS

2D	Two-Dimensional
3D	Three-Dimensional
CFD	Computational Fluid Dynamics
DNS	Direct Numerical Simulation
INSA	Institut National des Sciences Appliquées (National Institute of Applied Sciences)
LES	Large Eddy Simulation
LMFA	Laboratoire de Mécanique des Fluides et d'Acoustique (Acoustic and Fluid Mechanics Laboratory)
LS-PIV	Large-scale Particle Image Velocimetry
PIV	Particle Image Velocimetry
PTV	Particle Tracking Velocimetry
RANS	Reynolds-Averaged Navier-Stokes
SA	Spalart-Allmaras Turbulence Model
SGS	Subgrid-Scale
URANS	Unsteady Reynolds-Averaged Navier-Stokes

CHAPTER ONE

INTRODUCTION

1.1 OVERVIEW

The introduction of this thesis includes overall description of the study background, problem statement, research philosophy and scope, objectives, limitations, and significance. This chapter commences with an overview of the introduction to the topic under consideration, followed by a concise description of the research problem and the reasoning for the suggested solution based on the adopted approach of the study. The scope of the research is clearly defined, with a special focus on addressing the problem statement. The research objectives are then provided in a systematic and ordered way, offering a roadmap for the completion of the study. Finally, the chapter concludes with the outline of the thesis, offering a comprehensive summary of the organization and content of the research.

1.2 BACKGROUND OF STUDY

Water flowing in open channel networks, natural or man-made, invariably merges or divides in accordance to the physical configuration of such systems. Stream and rivers are formed by the combination of flows from branches or tributaries draining the upper reaches of contributing areas – in the literature, these meet at locations termed as junctions (Best & Reid, 1984). Split, divided or bifurcated flow can occur at many points in a river system due natural causes such as topographic and terrain changes resulting in the formation of distribution channels (commonly found in deltas). A diversion is a particular case of an asymmetric bifurcation where one channel remains on the original course; this is almost always manmade for several applications, as water supply, irrigation, and recently, reclamation of deltas (Dutta, 2017). In Figure 1.1, the West Bay engineered diversion channel off the Mississippi River is illustrative of a manmade version of naturally-occurring asymmetric bifurcations. Dutta (2017) reported that the distribution of water and sediments, denoted by Q and S respectively,

were often thought as being equal – however, investigations in the early part of the last century repudiated this view.



Figure 1.1 Mississippi River (Q, S), its West Bay diversion ($Q_{\text{side}}, S_{\text{side}}$) and downstream reach ($Q_{\text{main}}, S_{\text{main}}$). (Dutta, 2017)

Hager, (2010) specified that a channel bifurcation ensues with the incoming flow being divided into several downstream segments. The flow separation in such structures is larger and thus more intricate than the mainly one-dimensional flow in channel junctions (defined as the short distribution channel type). He also defined a T-bifurcation as ‘a straight through branch and a perpendicular lateral of equal or smaller width’.

1.3 PROBLEM STATEMENT

The hydrodynamic behaviour and sediment transport processes that occur when water flows in divided open-channel systems have been investigated since the early part of the last century. Such work was mainly triggered by the problems faced by engineers in confronting fluvial rivers, especially at bifurcations, as well as in constructing flow

diversions, junctions and intake structures. In most cases, the sediments at bifurcations and diversions impede engineering efforts when the flows become very turbulent and distorted, creating secondary circulatory patterns. The particles could be easily eroded and deposited at several locations within the bifurcated/diverted region.

As an example in Figure 1.2, deposited sediments can be seen forming a sandbar at the intake forebay of a water supply facility in the Ohio River (Neary et al., 1999). In addition, a significant amount of material is transported into lateral channel thus adversely affecting plant performance.

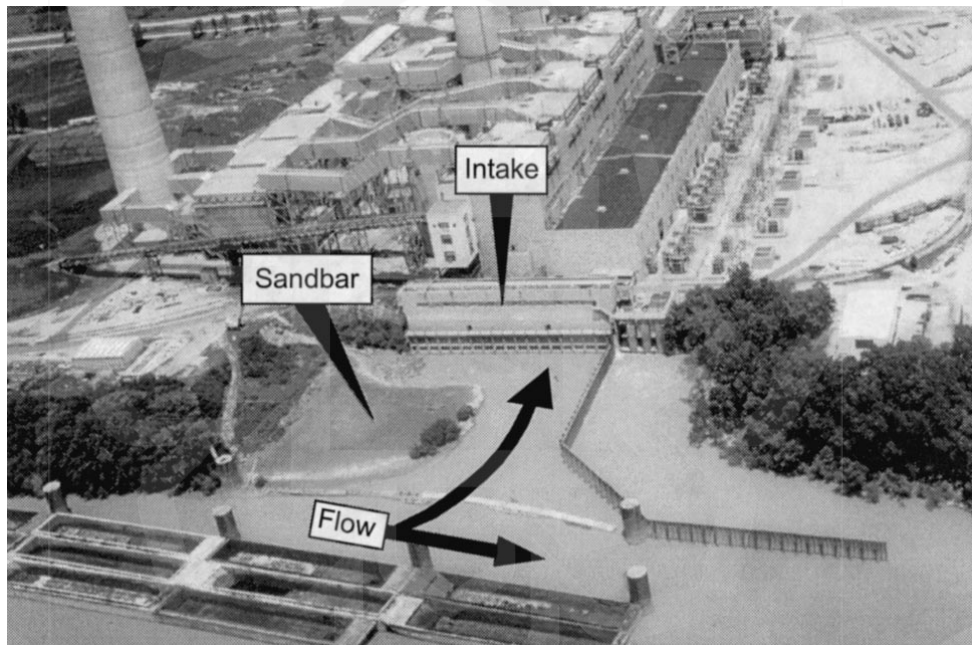


Figure 1.2 Sedimentation in the forebay of water supply intake adjoining Ohio River (Neary et al., 1999)

The underlying phenomenon in divided open channel flow is the Bulle-Effect, ‘discovered’ by H. Bulle from lab experiments conducted in 1926 (Dutta, 2017). As elaborated in Chapter 2 (Literature Review), the results based on [his observation of Figure 1.1 by Dutta, (2017)] indicated that the bed-load sediments are more inclined to flow into the lateral channel, even in conditions where the contrary is displayed by the distribution of water.

More recently over the past decade or so, researchers are exhibiting considerable interest in investigating the division of flows through open streets and roads during floods. Recently, more studies of bifurcated open-channel flow in other areas of river and stormwater hydrodynamics can be found with the advent of new experimental, analytical and computational techniques.

Based on experimental results, Neary et al. (1999) concluded that diversion flow is distinctly three dimensional and more readily diverted on channel bed/bottom. In addition, a disproportionate amount of bed sediments would enter diversion as compared to main channel. Such complex three-dimensional (3D) flow separation and patterns were also obtained from numerical solutions for steady, laminar (Neary & Sotiropoulos, 1996) and turbulent (Neary et al., 1999) conditions in rectangular open-channel T-junctions. It is noted here that the “closed semi-elliptic region developing along the upstream wall of the lateral branch” is inferred to be 2-dimensional.

Momplot et al. (2017) separated the flow behaviour in a right-angle diversion channel into 2D closed and 3D helix-shaped recirculation patterns which are distinct from each other. Computational simulations, supported by a limited number of experiments, indicate that these are dependent on and can be estimated by Froude number and the upstream or main channel aspect (flow depth to width) ratio. Thus, there is a need for more comprehensive experimental investigations to validate such findings and expand the body of work beyond equal-width channel bifurcations.

1.4 RESEARCH APPROACH AND SCOPE

From the review of existing literature published thus far on bifurcated open-channel flow (Chapter 2), the investigations are found to be limited by the sufficiency of experimental data to answer the following:

- Identification of different flow structures or recirculation patterns
- Characterization of recirculation types based on flow typology
- Empirical formulation of discharge distribution between main and lateral/branch channels

Therefore, the main approach and scope of this research is the conduct of experiments in laboratory settings. Firstly, these were executed to further examine the computational simulations and results published by the preceding work of Momplot et al. (2017). In continuation of investigations initiated and launched by Rivière et al. (2007) as well as Mignot et al. (2013), the same laboratory setup established at Laboratoire de Mécanique des Fluides et d'Acoustique (LMFA) of Institut National des Sciences Appliquées (INSA) in Lyon, France has been utilized after some reorganization of the configuration, for this purpose. A series of flume runs was then initiated with the original equal-width channels to validate findings of the previous study. The plan was to complete these first, followed by repeating the runs for the narrower branch width cases and conduct further rigorous examination of bifurcated flow. Originally, all experiments were scheduled over a three-to-four-month period beginning late January 2020, but due to the Covid-19 pandemic crisis and ensuing lockdowns, only the first set of experiments have been accomplished in March 2020. The second part was then successfully performed later in middle 2021.

1.5 RESEARCH OBJECTIVES

The main objectives of this research are:

1. To determine experimental techniques of visualizing flow structures and distribution in bifurcated open-channel flow
2. To investigate methods to identify three-dimensional flow structures in bifurcated open-channel flow
3. To improve empirical flow distribution relationships in open channel T-shape bifurcations
4. To establish the effect of reduced side branch width on empirical flow distribution relationships in open channel T-shape bifurcations

1.6 LIMITATIONS OF THE STUDY

This research was mainly focused on experiments conducted in a laboratory setting at LMFA, INSA Lyon. Thus, the scale and dimensions of the flume configured with a main and lateral/branch channels are rather limited in the representation of actual conditions. As described in Chapter 3, each channel of maximum 0.3-m width is made

of (smooth) glass with the main length measuring 4.9 m whilst the lateral/branch is 2.6 m long. Natural rivers and man-made conduits are customarily much larger in size and such constraints may cause significant bed/wall ‘disturbances’ although open channel flow is typically turbulent. Furthermore, the horizontal bed slope and by definition, right-angled T-bifurcation, further restrict the scope of practical real-world application.

The experimental runs were executed under the requisite steady-state conditions. In a closed system controlled and regulated by valves, even the constant pumping of water from the downstream main tank to the upstream chamber produced transients in the flow that agitated these supposedly time-invariant conditions. The resulting discharge fluctuations plus perturbations on the water surface affected precise measurements of flow rates and depths as well as hampered reliable identification of turbulent structures and patterns. Considerable effort was expended to minimize these issues.

The visualization techniques employed in the investigation of flow structures and patterns were somewhat restricted to readily-available material such as coloured paint or ink, thread, wood chips and the like – each was also deployed via testing of a few approaches and methods. An efficient and reliable set of procedures utilizing the most suitable material is essential for this purpose.

1.7 SIGNIFICANCE OF THE STUDY

The research contributes to the field of bifurcated open-channel flow by:

1. improving the characterization of flow structures and distribution, especially the most recent observations and interpretations of 2D and 3D patterns by Momplot et al. (2017);
2. scrutinizing the suitable methods and associated materials (such as coloured paint or ink, thread, wood chips) for identification of flow patterns;
3. providing an updated large database of at least 700 measurement sets to validate past and future computational investigations;
4. examining the effect of reducing the widths of the lateral/branch channel which is an identified research gap;

5. formulating new as well as improved empirical relationships of discharge distributions for both sub- and trans-critical flow regimes; and
6. determining the requirements for moving forward in such work and conducting further research.

1.8 THESIS STRUCTURE

This thesis is composed of five chapters. The first chapter is an introduction that comprises a general overview, background information, research objectives, a problem statement, and the scope of the study. The second is a literature review of the preceding research concerning firstly a historical perspective of research in bifurcated open-channel flow, especially the basis of the Bulle-Effect, followed by an in-depth examination as well as summary of past work on investigative approaches, flow description, structure and typology, discharge distributions. The research gaps and questions are also discussed in Chapter 2. The third chapter explains the methodology of the research, including a description of the experimental setup and material utilized for executing runs in the laboratory flume, variables and parameters tested, and methods for distinguishing flow patterns. The results, namely (i) observation and identification of flow structures, and (ii) distribution of flow discharges from main to lateral/branch channels, are subsequently presented and analyzed in Chapter 4 according to the stipulated research objectives. Finally, the fifth chapter summarizes the findings and conclusions based on these objectives, then ends with recommendations for future further studies.

CHAPTER TWO

LITERATURE REVIEW

2.1 HISTORICAL PERSPECTIVE

As described in Chapter 1 (Introduction), recent investigations of open-channel bifurcations, natural or otherwise, are concerned mainly with flow division, in terms of behaviour and quantification, within the main and branch waterways. This chapter is then a review of relevant literature conducted according to certain key topics as organized in the following sections.

2.1.1 The Bulle-Effect

In 1926, H. Bulle conducted a methodical study on the processes that distribute sediments travelling on channel beds of bifurcating channels at the Technical University of Karlsruhe, Germany (Bulle, 1926) (Dutta, 2017). The results of his laboratory experiments indicated that the bed-load sediments are more inclined to flow into the lateral channel, even in conditions where the contrary is displayed by the distribution of water. This phenomenon is widely known as the Bulle-Effect, described below by (Dutta, 2017) based on the following experimental setup (Figure 2.1) and results chart (Figure 2.2). It can be discerned in the latter (Figure 2.2) that the sediment discharge (S_{side}) entering the branch, bifurcation or intake is particularly greater than that in the main channel (S_{main}), even for $Q_{main} > Q_{side}$ flows (Dutta, 2017).

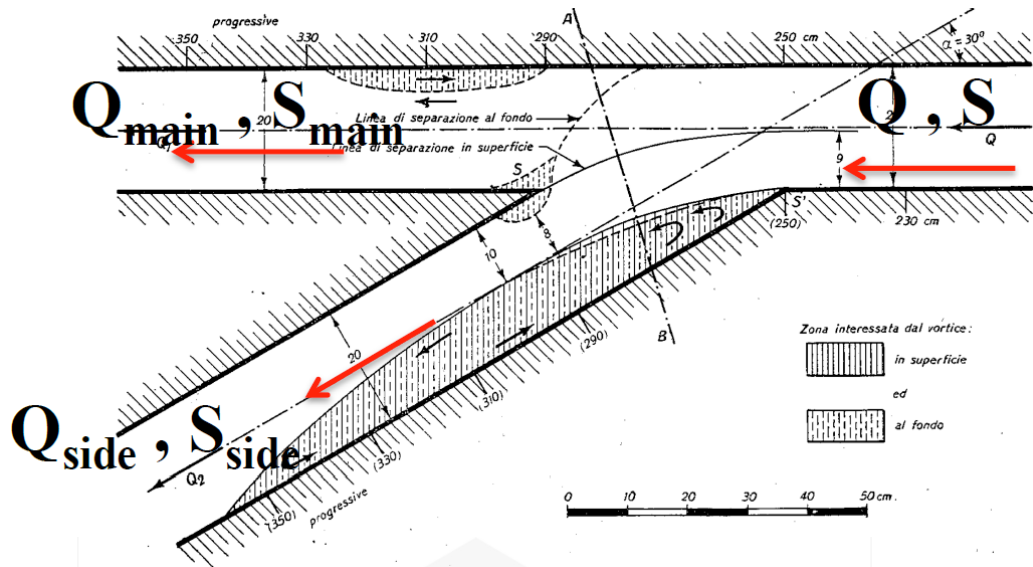


Figure 2.1 Schematic setup of Bulle's experiments (Dutta, 2017)

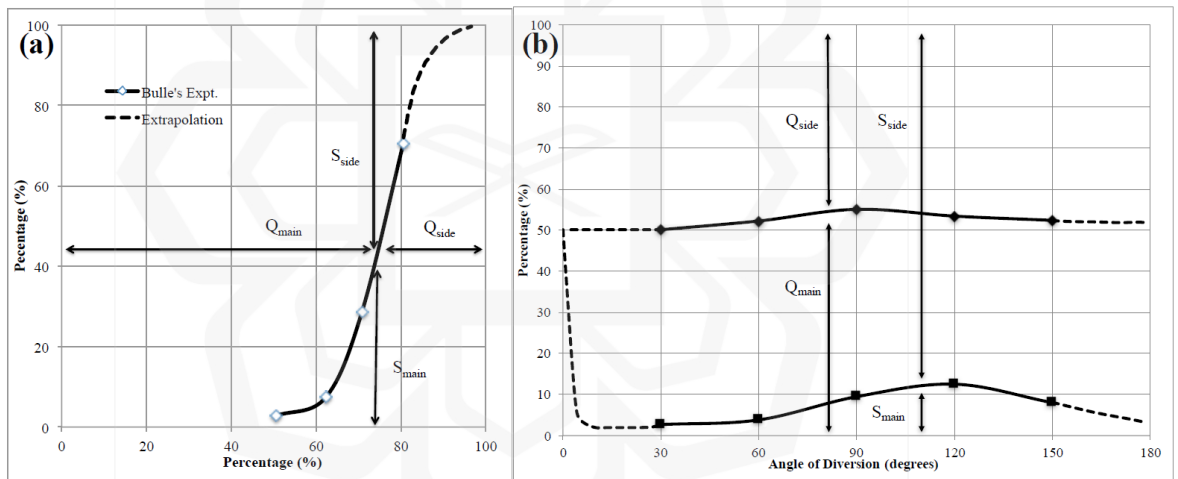


Figure 2.2 “Percentage of solid (S) and liquid (Q) discharges in the main (Q_{main} , S_{main}) and diverted (Q_{side} , S_{side}) channels for $Q_{total} = 0.005 \text{ m}^3/\text{s}$ (where $Q_{total} = Q_{main} + Q_{side}$). (a) Data is for a 30-degree diversion, where percentage of the total flow (Q) passing through each channel after the diversion is plotted on the x-axis, and the corresponding percentage of bedload has been plotted on the y-axis. (b) Data corresponds to experiments with different angles of diversion. Angle of diversion represents the angle between the center-lines of the main and the diverted channel. The solid line represents data from conducted experiments, and dashed line represents expected trends. Data reproduced from Bulle's experiments.” (Dutta, 2017)

Since then, many researchers have followed up with studies of natural bifurcations, namely Vogel (1934), Dancy (1947) and Lindner (1953), subsequently throughout the 1980s and 1990s until recently as highlighted in this review. Several aspects of the preceding work were assessed and discussed in terms of relevance to Bulle Effect, experimental setup, and computational effort.

In his PhD study, Dutta (2017) first investigated the Bulle-Effect phenomenon with high-resolution computer models utilizing CFD methods of Direct Numerical Simulations (DNS) and Large Eddy Simulations (LES). The results suggested that this effect occurs when most of the main channel flow (Reynolds number within 10 to 25,000 range) at its bottom is directed into the lateral branch, even during small fractions of the total discharge flowing into the bifurcation. This is aptly demonstrated by the DNS-simulated velocities in laminar (Bulle-Effect could be stronger than turbulent) flow conditions as illustrated in Figure 2.3.

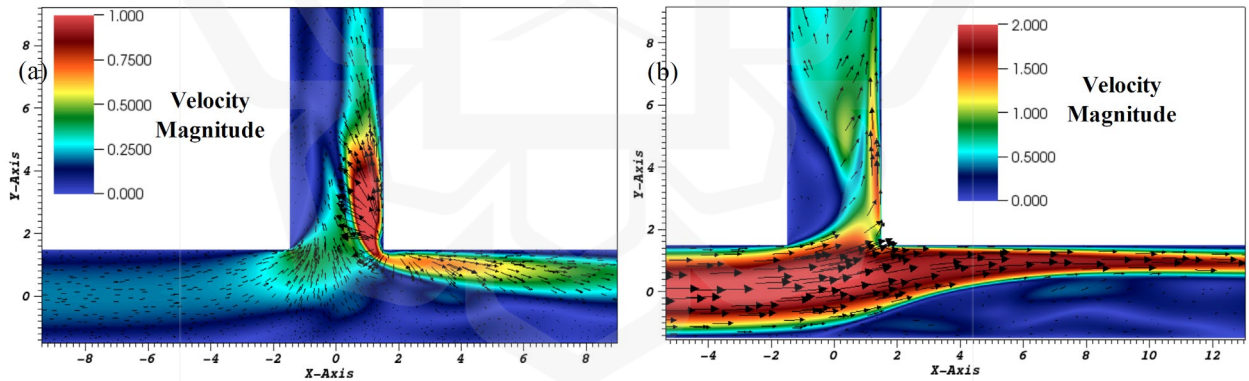


Figure 2.3 Velocity magnitude for $Re_{bulk} = 300$ and 50:50 flow split. (a) at 5 percent height from the bottom (b) at 90 percent height from the bottom (Dutta, 2017)

Dutta (2017) then ascertained that the Bulle-Effect can be adequately simulated in a Reynolds Averaged Navier-Stokes (RANS) based hydrodynamics model, compared with the much more numerically extensive (and expensive) DNS and LES models. A 3D version was then established to rerun Bulle's experiments in a channel with mobile (i.e. erodible) bed to explore the morphodynamics of bifurcations and

diversions. The results indicated that scouring occurred below the high-flow zones of the lateral channel, while sediments were deposited in its flow re-circulation area and around flow-stagnation zone at right corner of the diversion. He concluded that more simulations are needed to include bank erosion/failure and varying sediment input, as well as be extended to field-scale experiments.

2.1.2 Past reviews

The investigation of open channel junctions, bifurcations and side or lateral intakes is widely reported in the literature over the past few decades. Some of the major works are Bulle (1926), Grace & Priest (1958), Shettar & Murthy (1996), Neary & Odgaard (1993), Hsu et al. (2002), Ramamurthy et al. (2007), Mignot et al. (2013), Momplot et al. (2013), and Dutta (2017). Alomari et al. (2016) recently reviewed past studies to summarize relevant research work specifically on flow and physical characteristics as well as modelling of bifurcating open channel flow. The authors found that such literature mostly focused on channels with a right-angle branch and a rigid boundary, and therefore suggested further studies on flow characteristics in different branching angles and on mobile (sediment) beds. Specifically, they recorded in Table 2.1 a body of work covering substantial ranges of relevant properties, such as channel shapes, widths, slopes, branch angles, and flow types, of various extents. Besides additional physical features including bed roughness, the same authors then reviewed the published material in relation to the following flow characteristics:

- diversion discharge
- water depths, water surfaces, and hydraulic jumps
- velocity distribution and streamlines
- separation zones
- stagnation point
- contraction coefficient
- energy head, energy loss coefficient, and power

As for numerical and computational simulation of branching channel flow, Alomari et al. (2016) appraised models by other researchers, beginning with solutions

of one-dimensional (1D) mathematical formulations to more recent applications (due to rapid development in computer technology) in handling the full Navier-Stokes equations.

Thus, for the review in this research, the above is further assessed and refined to ascertain relevant properties for further investigation as follows:

- smooth rectangular main and branch channels
- fixed/rigid boundaries (bed and walls), as opposed to mobile/loose with sediments
- horizontal bed slopes
- right angle branch
- bifurcating flow
- subcritical flow in main channel

Table 2.1 Typical physical model properties in branching channel flow as reported by Alomari et al. (2016)

Authors	Main channel					Branch channel					Branch channel angle and location	Total discharge L/S	Type of upstream flow
	C.S ^I	L ^{II} m	W ^{III} m	d ^{IV} m	S ^V x10 ⁻³	C.S	L m	W m	d m	S x10 ⁻³			
Al Omari and Khaleel, 2012	R ^{VI}	10.0	0.3	0.45	0.25	R	2.0	0.15	0.3	1-2.5	30°, 60°, 90°, at 4.5	13 to 17.25	subcritical
Barkdoll et. al., 1999	R	24.0	1.5			R	2.44	0.61			90°, at 15.5	104	subcritical
El kadi Abderrezzak et al., 2011	R	4.91	0.3		0	R	2.61	0.3		0	90°, at 2.0	4 to 12	
Herrero Casas, 2013	R	9.0	0.2	0.35		R	2.5	0.2	0.35		90°, at 6.0	3.5 to 6.0	subcritical
Hsu et.al., 2002	R	12.0	0.147		0	R	4.0	0.147		0	90°, at 5.35	3.02 to 5.37	subcritical
Keshavarzi and Habibi, 2005	R	15.8	0.5	0.4	3	R					45°, 56°, 67°, 79°, 90°		subcritical
Kubit and Ettema, 2001	R	24.0	1.5			R	2.44	0.61			90°, at 15.5		subcritical
Lama et al. 2002	R	8	0.78		0	R	2	0.045		0	30°, at 3.0	5.1 to 9.4	subcritical
Maghadam et.al., 2010; Karami-Moghaddam et.al., 2011; Bejestan et.al., 2013	T ^{VII}	8	0.225	0.7		R	5	0.2	0.7		30°, at 5.5	5 to 35.2	subcritical
Mignot et al., 2014	R	4.9	0.3	0.2	0	R	2.6	0.3	0.2	0	90°, at 2.0	4.0	subcritical
Neary and Odgaard, 1993	R	18.3	1.2	0.61		R	1.2	0.61	0.61		90°, at 13.1	Q _I =0.50	subcritical
Omidbeigi et. al., 2009	R	18	1.0	0.9		R	2	0.4	0.4		90°, at 11.43	58	subcritical
Ramamurthy et. al., 2007	R	6.198	0.61	0.305		R	2.794	0.61	0.305		90°, at 2.794	46 to 47	subcritical
Ramamurthy and Satish, 1988	R	5.454	0.254	0.432	0	R	2.6	0.254	0.432	0	90°, at 2.6		subcritical
Riad, 1961	R	19.9	0.8			R	10	0.5			45°, at 8.2	30.1 to 111.3	subcritical
Yonesi et al., 2008	R	30	1.5		5	R	2.7	0.6		0	60°, at 23	45 to 90	subcritical

Where: ^I= cross section; ^{II}, ^{III} and ^{IV} = length, width, depth in meter, ^V = bed slope, ^{VI} and ^{VII} = rectangular and trapezoidal cross section.

2.2 APPROACHES AND METHODS OF INVESTIGATION

From the review of work by others in Table 2.1, investigations on bifurcated open-channel flow over the years began mainly with experimental [Krishnappa & Seetharamiah (1963), Law (1965), Lakshmana Rao & Sridharan (1967), Neary & Odgaard (1993)] then proceeded recently to more emphasis on computational methods.

According to Rivière et al. (2007), the validity of assumptions, namely one-dimensionality and hydrostatic pressure distribution, made in analytical approaches for describing such divided flows is doubtful resulting from the experimental observations of contracted recirculation zones. Real practical conditions such as in flooded channels and streets would also undermine these models which were developed based on mass, momentum and energy conservation [Gurram et al. (1997) & Shabayek et al. (2002)]. These were also focused on merging or combining incoming flows, rather than bifurcating. Other researchers like Hsu et al. (1998) then established empirical formulations with factors to represent various physical features (such as contraction coefficient) which are heavily influenced by experimental conditions. Earlier in this chapter, the works of Neary and Odgaard, (1993) and Neary et al. (1999) revealed the three-dimensional behaviour of bifurcated open-channel flow thus nullifying the earlier 1D postulation.

Furthermore, studies on discharge distributions in bifurcations have been hampered by the unquantified flows when split into downstream channels, specifically in earlier 1D investigations due to the inadequate number of equations based solely on the utilization of the momentum balance (Taylor, 1944). Rivière et al. (2007) concluded that this approach is also restricted by the occurrence of intense 3D recirculation patterns in the lateral channel which permit only the estimation of flow ratios based on depth ratios, the latter requiring supplementary knowledge, equation, or ‘closure’ [Law & Reynolds, (1966), Ramamurthy et al. (1990), Hsu et al. (2002)]. Evidently, such ‘closure’ based on energy deficits is not practical due to very minimal magnitude of losses occurring within turbulent flows in ‘smooth’ channels (Hager, 1992).

Almost all researchers reviewed derived the set of parameters relevant to the investigation of bifurcated flow from dimensional analysis. Rivière et al. (2007)

reasoned that this approach could also lead to the formulation the required ‘extra’ equation to complete the discharge distribution or ratio analysis. However, the dependence of these equations on flow regimes in the channels, especially at downstream reaches and boundaries, severely limits their range of applicability. Some investigators relied on the occurrence of choked flow regions, where the overall subcritical conditions were subjected to local Froude number reaching 1 and beyond, to reduce the variable or parameters of consideration. For example, Krishnappa & Seetharamiah (1963), developed empirical equations from dimensional analysis based on upstream Froude number, while Lakshmana Rao & Sridharan (1967), did the same as a function of the downstream value.

Computer model simulations offer new understanding of the intricate turbulent hydrodynamics exhibited by zones of flow division, separation-reversal, vortices, singular points within the bed-shear stress vector field of lateral intakes which might not have been possible from experimental testing alone (Neary et al., 1999). A few years earlier, the same group of researchers, Neary & Sotiropoulos (1996), completed numerical runs of bifurcating flows under steady, laminar conditions. The major finding of recirculatory flow patterns in the branch channel and conclusions then prompted such work to be continued in (Neary et al., 1999). It is then that a “three-dimensional (3d) numerical model for predicting steady, in the mean, turbulent flows through lateral intakes with rough walls is developed, validated, and employed in a parametric study. The method solves the Reynolds-Averaged Navier-Stokes (RANS) equations closed with the isotropic κ - ω turbulence model of Wilcox (Wilcox, 2008), which resolves the near-wall flow and accounts for roughness effects in a straightforward manner.”

With the advent of advanced computational fluid dynamics (CFD) techniques in recent years, research on flows in open-channel junctions are revitalized with such model studies to simulate the complex 3D turbulent flow behaviour and sediment transport processes. As for numerical and computational simulation of branching channel flow, Alomari et al. (2016) appraised models by other researchers, beginning with solutions of one-dimensional (1D) mathematical formulations to more recent applications (due to rapid development in computer technology) in handling the full

Navier-Stokes equations. The latest turbulence-closure models of 2D and 3D Reynolds-averaged Navier–Stokes (RANS) equations include the following:

- $\kappa\text{-}\omega$
- $\kappa\text{-}\varepsilon$
- $\kappa\text{-}\varepsilon$ along with Renormalization Group (RNG)
- Boussinesq
- Reynolds stress (RSM)
- Spalart-Allmaras (SA) (Mignot et al., 2013)
- Large Eddy Simulation (LES)
- Direct Numerical Simulation (DNS)

In addition, the more advance turbulence models include Spalart-Allmaras (SA) (Mignot et al., 2013), Large Eddy Simulation (LES) and Direct Numerical Simulation (DNS). The implementation of DNS, in most cases, is optimum to accurately model the whole range of turbulent eddies, only to be hindered by hardware limitations and time constraints. For Dutta (2017), the full Navier-Stokes equations are numerically ‘solved’ as follows:

“The model described in the previous sections was implemented on the massively parallel open-source incompressible Navier-Stokes solver Nek5000 (Fischer et al., 2008). Nek5000 uses spectral-element method for spatial discretization (Deville et al., 2002), with higher-order orthogonal Legendre polynomial basis on Gauss-Lobatto-Legendre (GLL) nodes. The temporal discretization in Nek5000, uses a semi-implicit scheme that is a mix of third-order Backward Difference (BDF3) and Extrapolation (EXT3) schemes. In this section, different components of the particle model would be tested to check the veracity of the model.”

“Depending on the Re_{bulk} (bulk Reynolds number), the simulations required anywhere between 37.5 million to 240 million computational

grid points. Thus the simulations were conducted on the peta-scale High Performance Computing (HPC) system Blue Waters, NCSA, UIUC.”

This indicates the massive resources required to effectively model turbulence for the required Re_{bulk} range of 10 to 25,000. The numerical and CFD simulations are further impeded by the necessity to simulate sediment-laden flows utilizing a semi-implicit Lagrangian particle tracking algorithm. After further testing at all levels of resolution and complexity, Dutta (2017) then discovered that 3D turbulent processes in bifurcations can be effectively represented by RANS formulation, in lieu of the DNS and LES models.

Other researchers managed to overcome the ‘resources-problem’ by reducing or simplifying the Navier-Stokes governing equations as highlighted above by Alomari et al. (2016). In particular, Li & Zeng (2009) utilized a hybrid model of 3D RANS and LES, in which complex turbulence and vortices at the 90° junction were simulated with the latter, while the former addressed the other regions in both channels.

In their investigation of turbulent structures at a right-angle junction resulting from divided open channel flows of equal widths, Momplot et al. (2017) conducted CFD simulations with ANSYS Fluent to solve RANS equations using a finite volume method. The adopted numerical scheme was a Reynolds stress model (RSM) as a complete Reynolds stress closure developed by (Launder et al., 1975), with scalable near-wall treatment proposed by (Grotjans et al., 1998). It was found that the computed results (of just solving RANS) agree quite well even with a basic mesh of 700,000 cells – this model setup is described further in research methodology. This computational approach allowed the investigators to simulate numerous flow cases (at least 16) that cover a substantial range Froude number and aspect ratio, which would not have been practical even possible to conduct as experiments. From plotting of the results (described in more detail later in this chapter), Momplot et al. (2017) derived an equation as the linear boundary that separates the two different recirculation structures or patterns.

2.3 FLOW DESCRIPTION

Experimental works dedicated to better understanding the flow patterns within open-channel bifurcations have mostly considered T-shaped junctions of simplified rectangular, smooth, and horizontal channels (Ibrahim et al., 2020). Neary & Odgaard (1993) and Neary et al. (1999) described the 3D flow structure within the bifurcation in the subcritical regime, including the recirculation zone [Kasthuri & Pundarikanthan (1987), Momplot et al., (2017)] facing the accelerated zone in the side branch separated by a curved shear layer described by Mignot et al. (2014). In addition, Lakshmana Rao & Sridharan (1967) revealed different possible flow patterns in the “transcritical flow” regime, which were later explained by Rivière et al. (2014). When the flow is subcritical in the entire bifurcation, the maximum Froude number in the side branch is located just downstream from the intersection in the *vena contracta*, i.e. the section with the maximum recirculation width (Figure 2.4).

If the water depth in the downstream channel increases or that in the side branch decreases (owing to a modification of the downstream controlling boundary conditions), the relative discharge entering the side branch increases. Consequently, the Froude number in the *vena contracta* (Figure 2.4) increases and eventually reaches 1 (choked flow section). Thereafter, the flow accelerates downstream from the critical section, as the flow section increases with decreasing recirculation zone width. This combination of subcritical and supercritical flow regions was labelled as “transcritical” by Rivière et al. (2014). Depending on the downstream conditions and length of the branch channel, hydraulic jumps may form further downstream, leading to two characteristic configurations of the transcritical regime shown in Figure 2.4, based on Figure 1 of Rivière et al. (2014).

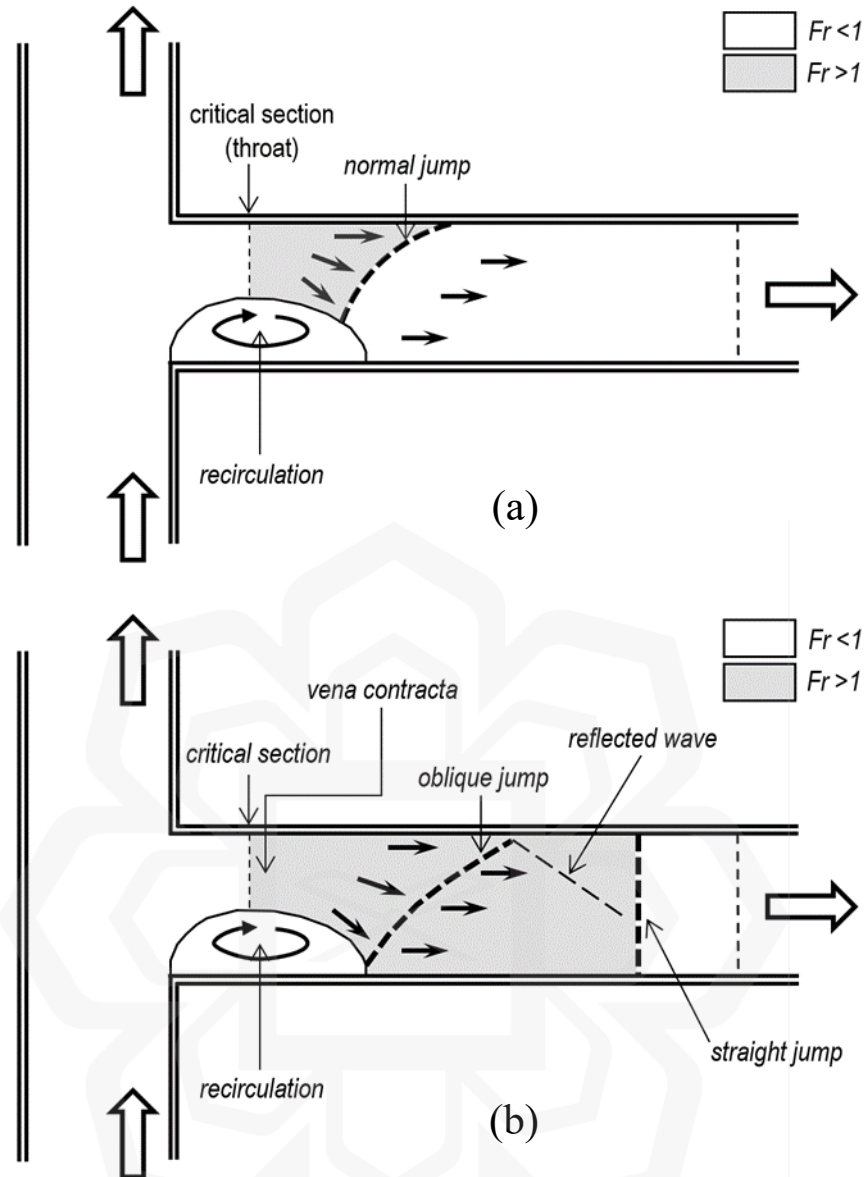


Figure 2.4 Drowned (a) and free (b) recirculation patterns in equal-width T-shape bifurcation as manipulated by downstream controls, based on Figure 1 of Rivière et al. (2014).

First as depicted in Figure 2.4(a), for a water depth at the downstream end of the branch only slightly lower than for the appearance of the critical section, a hydraulic jump appears in the diverging part of the side branch flow, namely in the region where the width of the recirculation decreases downstream of the *vena contracta*. In such a case, the hydraulic jump does not cover the entire width and is restricted to the main flow of the lateral/branch channel, that is, between the sidewall and the shear layer bounding the recirculation zone. In this first configuration, named “drowned

recirculation” by Rivière et al. (2014), the downstream boundary condition of the branch influences both the shape and depth of the recirculation.

Next in Figure 2.4(b), as the downstream depth in the branch decreases further, the hydraulic jump moves downstream from the end of the recirculation zone, and then occupies the entire lateral/branch channel width. In this second configuration, named “free recirculation,” no downstream control occurs in the branch because the flow along a certain channel section is in supercritical regime over the entire width. As discussed later, these regime variations (within the overall subcritical conditions in both main and lateral/branch channels) have been found to affect the dependence of flow division or distribution at the bifurcation on the boundary condition or outlet of the side branch.

A specific criterion or threshold for the transition from the subcritical to transcritical regime has not yet been definitively provided thus far in the literature. Ramamurthy & Satish (1988) did consider, referring to data from Lakshmana Rao & Sridharan (1967), that a choked flow occurs in the *vena contracta* when the downstream side-branch Froude number reaches 0.3 to 0.35, causing a transcritical flow. Later, Rivière et al. (2014) showed that the transition from drowned to free-recirculation transcritical sub-regimes occurs for a downstream side-branch Froude number reaching 0.65.

2.4 TYPOLOGY OF FLOW STRUCTURES (PATTERNS)

Among the earliest researchers studying flow structures and patterns in divided open channels is Vincent S. Neary as reported in his 1992 MSc study, “Flow Structure at an Open Channel Diversion”, at the University of Iowa. This was followed by experiments conducted Neary & Odgaard (1993) resulting in the finding that diversion flow is distinctly three-dimensional and more readily diverted on channel bed/bottom. In addition, a disproportionate amount of bed sediments would enter diversion as compared to main channel. Such complex three-dimensional (3D) flow separation and patterns were also obtained from numerical solutions for steady, laminar (Neary & Sotiropoulos, 1996) and turbulent conditions in rectangular open-channel T-junctions. It is noted here, in that as discerned from Figure 2.5, the “closed semi-elliptic region developing along the upstream wall of the lateral branch” (Zone A) was inferred to be

2-dimensional. Later in their series of research, Neary et al., (1999) also described in Figure 2.5, the general pattern of a steady subcritical three-branch bifurcation as a *three-dimensional* recirculating zone that developed in the lateral branch with secondary flows occurring in both outlets. They concluded that the second flow pattern was definitely unlike the 2D closed region observed earlier on the upstream wall, i.e. its structure was a series of three-dimensional helix-shaped streamlines along the opposite downstream and left walls of both lateral and main channels., respectively.

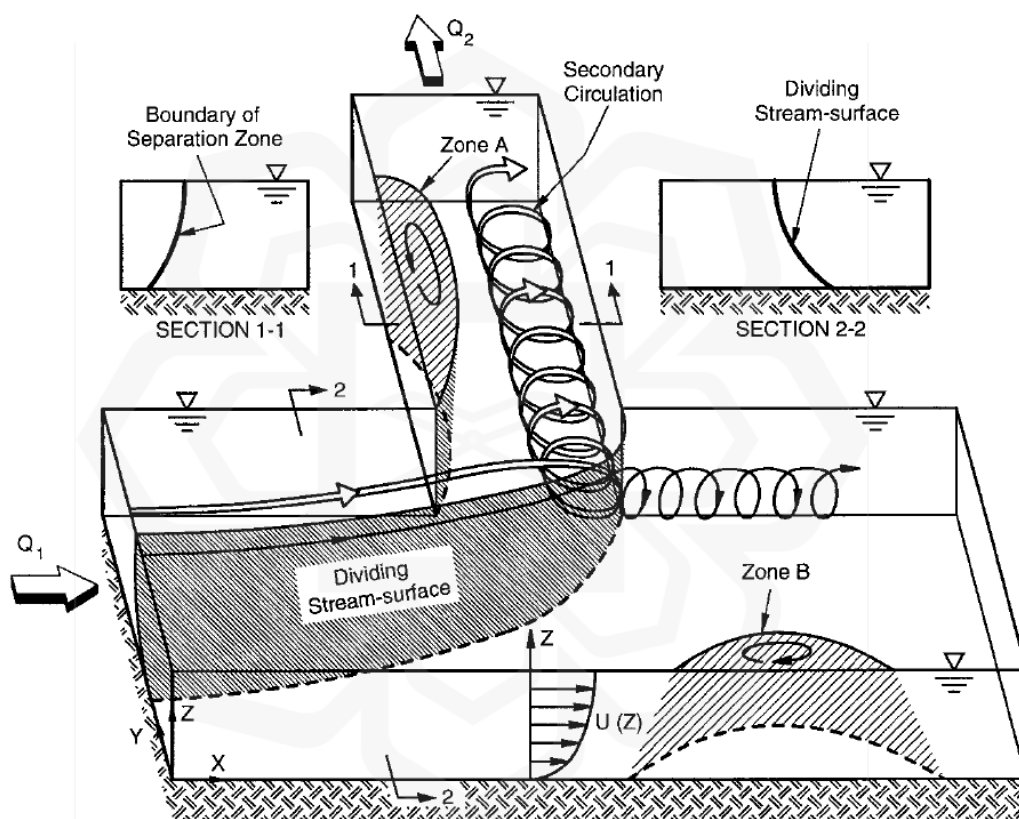


Figure 2.5 3D flow patterns in a lateral intake (Neary et al., 1999)

For rectangular open-channel bifurcations of various angles δ , Hager (2010) observed intricate secondary flow patterns with powerful bed currents in the lateral/branch as illustrated in Figure 2.6. In a similar manner to the Bulle-Effect, these are thought to drive much of the loose bed material into this channel, and thus Hager (2010) concluded that intakes should be positioned in the main channel downstream of the bifurcation to minimize sediment clogging. He also reported and presented the flow

patterns as influenced by the discharge ratio (R_q) in Figure 2.7, based on the depictions of Mock (1960). It is noted here that Hager (2010) defined the term T-bifurcation as “a straight through branch and a perpendicular lateral of equal or smaller width”.

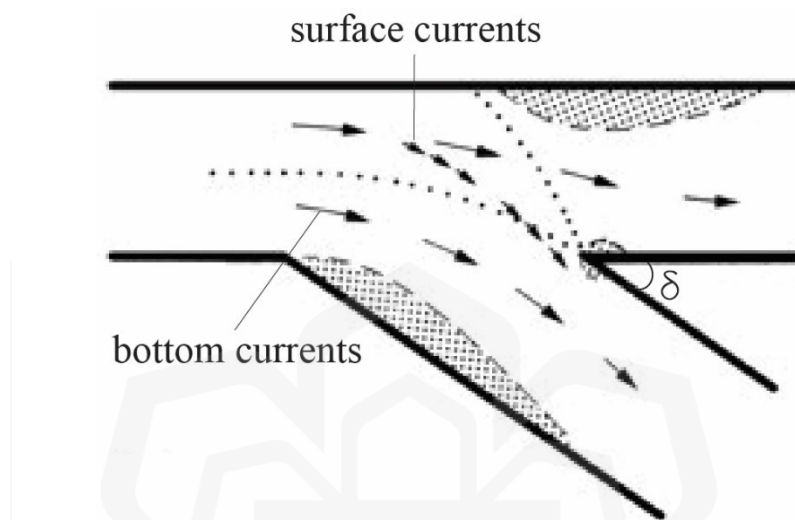


Figure 2.6 Surface & bottom currents in bifurcated open-channel flow according to Hager, (2010)

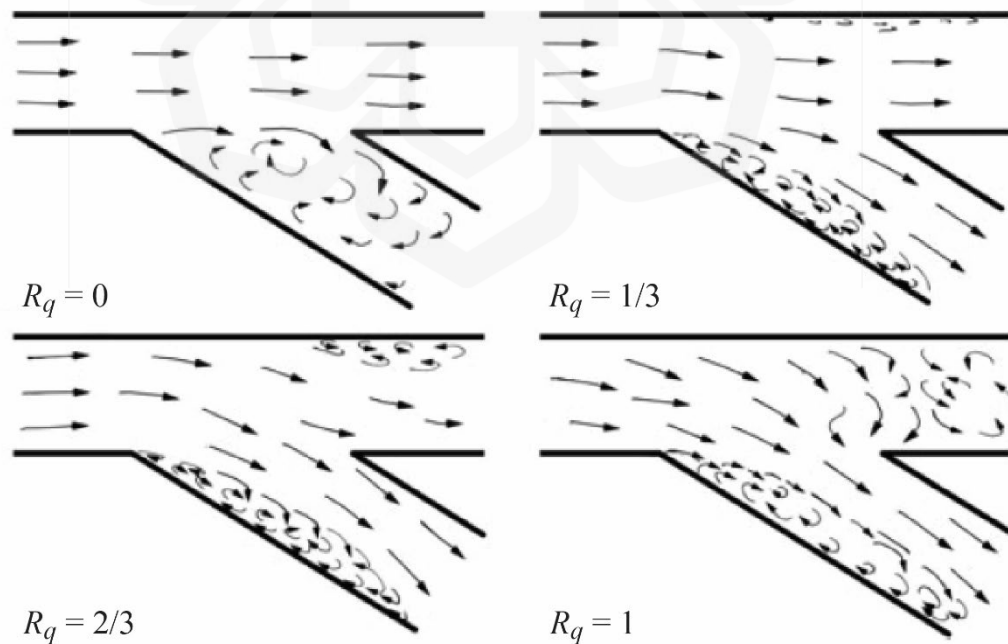


Figure 2.7 Flow patterns in bifurcated open-channel flow (Hager, 2010) based on Mock (1960)

Based on a study on flow typology (types) within bifurcations by Momplot et al., (2017), it is highlighted that the primary issue of past research has been to quantify the delivery of upstream (incoming) discharges into downstream (outgoing) branches. Furthermore, there is also a need to recognize the behaviour of detailed flow structures or patterns at such open-channel intersections or junctions.

In the computational modelling work of Li & Zeng (2009), a “recirculation bubble” (with at least one eddy) emerged downstream of the ninety-degree flow junction after flow was directed into lateral channel: The discharge ratio of main to branch seems to affect the size of this bubble. From studies with sediments on mobile beds, the locations of secondary flows in both main channel and branch were apparently determined by scouring and sedimentation patterns (Herrero et al., 2015). Nevertheless, the “numerical results confirm the flow patterns described in the experimental analysis of Neary et al., (1999), and are compared with the results in Bulle (1926) or Barkdoll et al. (1999) studies (Heer & Mosselman, 1926).” 3D computational models were considerably better than 2D in simulating the complex flow structures observed in experiments.

As such, Momplot et al., (2017) studied these two different recirculation structures in the lateral branch of a dividing open channel flow with identical downstream weirs by conducting computer simulations and lab experiments. They revealed that it is possible to predict the occurrence of each form based on flow conditions, namely the Froude number, and geometry, represented by the aspect ratio in the upstream channel of the bifurcation. In addition to the standard 2D closed structures, the authors/researchers successfully simulated complex 3D helical patterns and recreated the “Bulle-Effect” where a significant portion of the upstream flows near the main channel bed is diverted into the diverging (lateral) branch within the recirculation zone. These are illustrated in Figure 2.8.

(a)

(b)

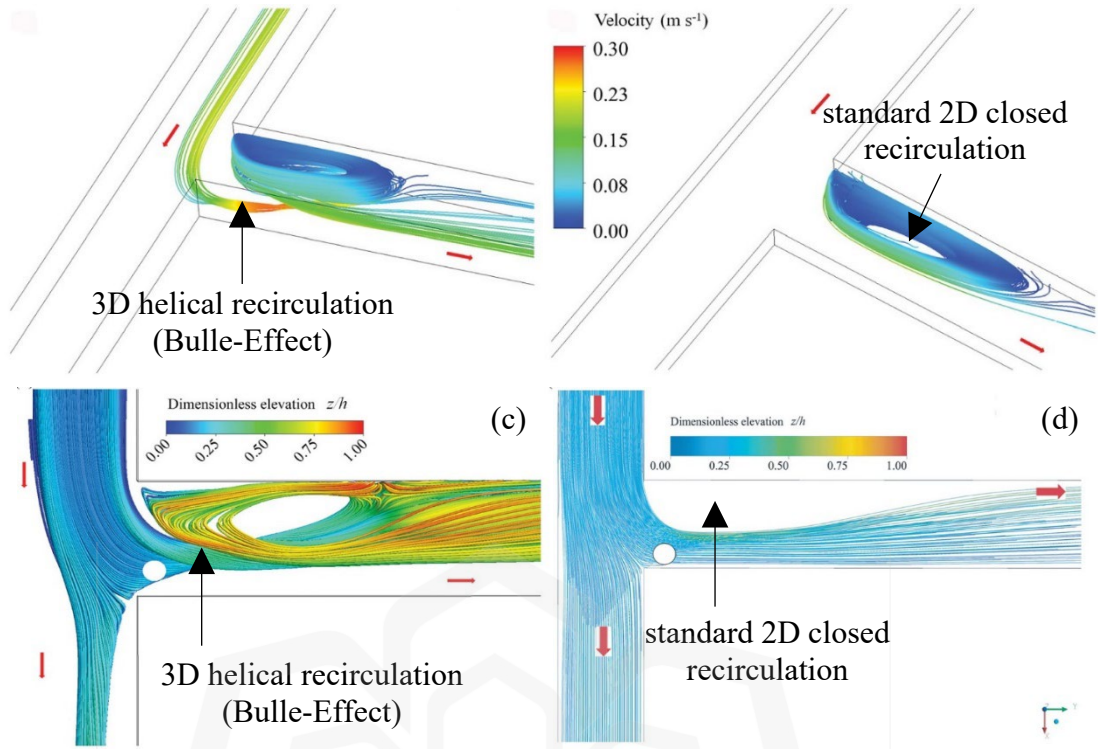


Figure 2.8 Comparison of computed streamlines plotted for F1 [(a), (c)] and F2 [(b), (d)] experimental runs by (Momplot et al., 2017)

In concluding their work, Momplot et al., (2017) reported that the typology of the two flow structures in open-channel bifurcations can be described by the ratio of centrifugal and pressure forces, namely the square of upstream Froude number $(F_{bx})^2$ and upstream flow aspect ratio (b_x/h_u) . The assumptions of a hydraulically smooth turbulent flow regime with equal channel widths and weir crest heights at both outlets must be emphasized here. “This typology defines two clear regions corresponding to each recirculation structure occurrence separated by a linear boundary”, i.e.;

$$(F_{bx})^2 = 0.047 \frac{b_x}{h_u} - 0.094 \dots\dots\dots (2.1)$$

[equation 6 of Momplot et al. (2017)]

This equation is fitted based the experimental data for the two regions and plotted in Figure 2.9.

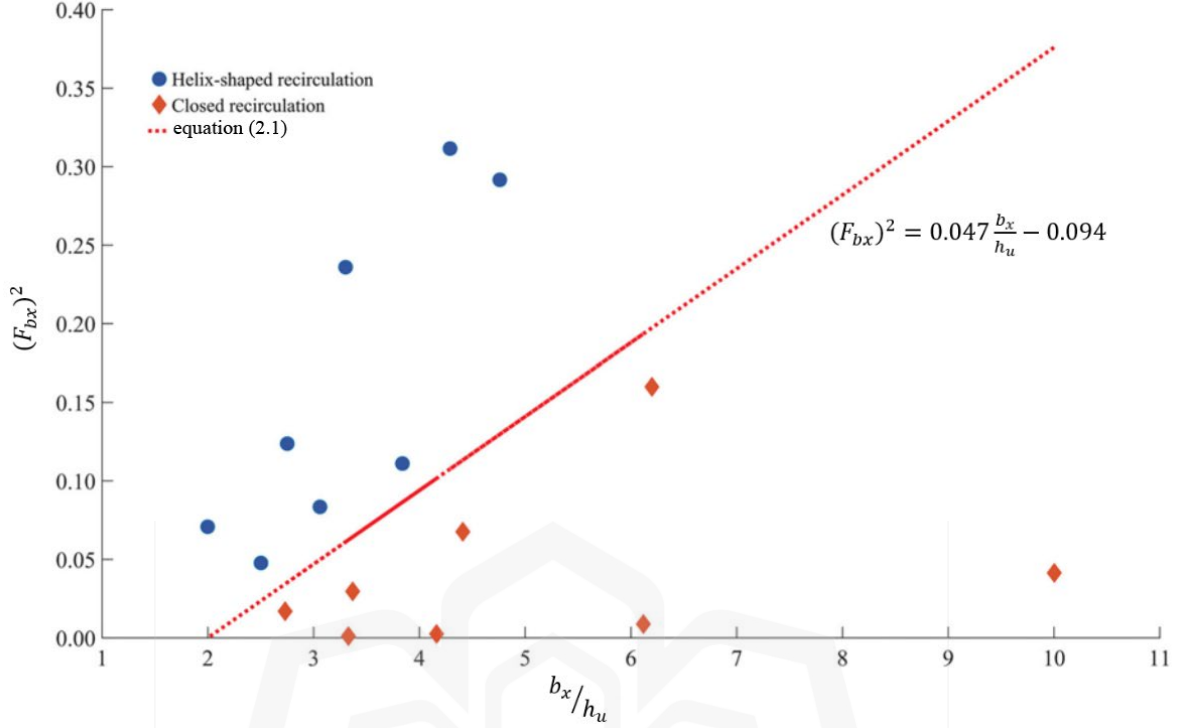


Figure 2.9 Flow typology for the recirculation zone in the lateral/branch channel. The red-dashed line represents the estimated boundary between 2D closed and 3D helical patterns as proposed by Momplot et al. (2017)

For further study, Momplot et al., (2017) suggest the investigation of other flow characteristics that may affect the centrifugal forces, specifically bed roughness and different channel widths. “Further work may extend the present results to sediment transport studies. Indeed, while the classical, closed recirculation zones are usual locations for silting to occur, for example in braided rivers or in sewer systems, the upward, helical shaped motion reported in this paper may induce different sediment transport patterns.”

2.5 CORRELATIONS OF DISCHARGE DISTRIBUTION

In a review by Alomari et al. (2016), it is well understood that the parameter R_q , i.e. the ratio of discharges, is often utilized to analyze dividing open channel flows. “This ratio depends on many factors, such as the Froude number upstream and downstream of the main channel and in the branch channel (F_u , F_d , and F_b , respectively), the water depths upstream and downstream in the main channel and in the branch channel (y_u , y_d , and

y_b , respectively), the bed slope (S), the branching angle (θ), the bed width ratio, the branch channel to main channel bed width (B_r), the bed roughness, and, if there is a side weir, the shape of the crest and weir height.”

Apart from the above, a comprehensive analysis, founded on the basic principles of mass and momentum conservation, of intersecting open channel flows was presented by (Rivière et al., 2011). This work actually assesses subcritical flow in four branch intersections – however, its basic concepts are based on the investigation of three-branch intersection flows.

It is reported by the several researchers [(Rivière et al., 2011) amongst them], that Neary & Odgaard (1993) is the main reference for investigating the general features of steady subcritical flow in a bifurcating network of three open channels. With the identification of inflow distribution between divided channels as an extra unknown, the principles of mass conservation and momentum balance alone are unable to solve the problem. The method, however, allows the recognition of another dimensionless parameter, specifically the ratio of Froude numbers, to characterize the flow conditions. Additional equations specified at the boundaries, derived from momentum balance at the intersection and dimensional analysis, are then needed to provide empirical closure and full resolution.

Rivière et al. (2007) investigated the general problem of free-surface flows divided through an intersection of four equal-width open channels in a laboratory setting, which is the foundation for Momplot et al. (2017) and this study. For subcritical (Froude number not exceeding 0.8) conditions in the 3-branch bifurcation configuration, the following equations (with notations adjusted for consistency) were established:

$$h_d = C_x + h_{crit,xo} \left[1 + 0.793 \left(\frac{C_x}{b_x} \right) \right] \dots\dots\dots (2.2a)$$

similarly, for lateral or branch channel

$$h_l = C_y + h_{crit,yo} \left[1 + 0.793 \left(\frac{C_y}{b_y} \right) \right] \dots\dots\dots (2.2b)$$

$$F_{Cxo} = \frac{h_{crit,xo}}{C_x^{3/2}} \dots\dots\dots (2.3)$$

$$R_{q-sym} = 0.5 - 0.04675F_{Cxo} \dots\dots\dots(2.4)$$

$$R_{q-sub} = R_{q-sym} + 0.358(1 - R_C)F_{Cxo}^{-0.77} \dots\dots\dots(2.5)$$

Equations (2.2a) and (2.2b) are the stage-discharge relationships or rating curves for the downstream and lateral/branch channels, respectively, derived and calibrated empirically based on weir heights (Rivière et al., 2007). From the earlier work of Rivière et al., 2007, the empirical closure for ratio of discharges in 3-branch dividing open channel flows (subcritical everywhere) was then formulated.

Further in their series of investigation, Rivière et al. (2014) subsequently observed the drowned and free recirculation transcritical flow patterns within the bifurcation (side branch) as highlighted previously. Specifically, they reported that when the side branch flow reaches its maximum value [‘free’ in Figure 2.4(b)], the discharge distribution becomes independent of the downstream boundary condition at the outlet. Conversely, in the drowned recirculation configuration described [Figure 2.4(a)], the supercritical flow region does not cover the entire side branch width, and a downstream control by the weir or fixed water depth at the end of the channel occurs via the recirculation zone. Therefore, the discharge distribution at the bifurcation is affected by the outlet or downstream boundary of the side branch.

A definite criterion or threshold for the transition from the subcritical to transcritical regime has not yet been provided. Ramamurthy & Satish (1988) did consider, referring to data from Lakshmana Rao et al. (1968), that a choked flow occurs in the *vena contracta* when the downstream side-branch Froude number reaches 0.3 to 0.35, causing a transcritical flow. Later, Rivière et al. (2014) showed that the transition from drowned to free-recirculation transcritical sub-regimes occurs for a downstream side-branch Froude number reaching 0.65.

In addition, empirical correlations for the prediction of discharge distribution in the downstream branches have been derived by several authors for flows in T-bifurcations of 90° angle (among other angles), both in the subcritical and transcritical regimes. These are described in the following sub-sections.

2.5.1 Discharge distribution correlations in subcritical regime

In previous investigations of subcritical flow regimes (also noted as “submerged” by several authors), Lakshmana Rao et al. (1968) measured the discharge distribution in a bifurcation with the side channel width half of the main branch, for a range of Froude numbers in the downstream and side branches. The authors observed that the discharge distribution to the side branch increases as the Froude number decreases in the main stream (owing to decreased centrifugal forces applied on the flow in the intersection) or increases in the branch (owing to a lower downstream control in this side branch). Hager (1991) and Hager (2010) proposed an empirical correlation fitted to these data, based on governing non-dimensional parameters derived from a dimensional analysis. However, this correlation was restricted to a branch-width ratio of 0.5. Therefore, the first research gap identified for the present work is the need to extend this discharge distribution database to several width ratios lower than one and evaluate the applicability of the Hager (1991) fitting correlation.

Rivière et al. (2007) conducted experiments for an equal-width T-shaped bifurcation (90° angle) in subcritical flows. The authors proposed an empirical discharge distribution correlation that fits well with their measurements, based on governing nondimensional parameters, but dissimilar to Hager (1991). However, this correlation is restricted to bifurcations of equal widths. Moreover, the selected parameters are constructed using the experimental weir crest heights as downstream boundary conditions, whereas most bifurcations encountered in the riverine environment do not include such downstream controls. This limits the applicability of Rivière et al. (2007) empirical correlation. The second research gap requirement is then to modify the nondimensional parameters and propose an empirical discharge distribution correlation that is adapted to natural or man-made bifurcations, based on downstream water depths in both divided channels. In addition to the equal-width case, this proposed correlation shall be extended to several variable branch width ratios.

Note that other bifurcation data in subcritical regimes were provided in horizontal equal-width T-bifurcations, namely by Law (1965) when studying the flow pattern and discharge distribution, by Mignot et al. (2013) when investigating the impact of obstacles in the intersection on the discharge distribution, and by Li et al.

(2020) when examining the effect of vertical distortion on bifurcation flow characteristics. This information is available and included for the evaluation of the newly developed discharge distribution correlations.

2.5.2 Discharge distribution correlations in transcritical regime

From a much earlier publication, Lakshmana Rao & Sridharan (1967) investigated to predict the discharge distribution in free-flow conditions, designated here as the free-recirculation transcritical regime. The authors studied five bifurcation angles and several ratios of side to main branch widths for each angle. The dataset of the 90° T-shape configuration considered four width ratio (R_b) values, namely $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and 1. They indicated that the discharge distribution to the side branch increased when the main channel Froude number decreased, thereby lowering the centrifugal force of the flow within the intersection. Moreover, the authors revealed that increasing the width ratio increased the discharge distribution to the side branch for all Froude number values in the main stream. They proposed a correlation for the discharge distribution as a function of the Froude number in the downstream branch based on two coefficients K and x , evolving as a 3rd order polynomial in relation to the branch-width ratio. Rivière et al. (2014) tested the validity of this correlation for an equal-width T-shaped bifurcation and found that the coefficients recommended by Lakshmana Rao & Sridharan (1967) are not optimal. Rivière et al. (2014) proposed to (1) slightly adjust the two coefficients of the correlation; and (2) change the location of the Froude number measurement in the downstream branch to a position one branch width downstream from the intersection instead of the entrance of the downstream branch [as proposed by Lakshmana Rao & Sridharan (1967)], where the flow is not fully one-dimensional. In addition, Rivière et al. (2014) reported that this correlation is adapted for both patterns of the transcritical regime: free flow and drowned recirculation. Therefore, it is necessary that this present research extends the Lakshmana Rao & Sridharan (1967) correlation to channel width ratios differing from those previously studied by the authors. In addition, the work continues with verification of whether the slight modification of the coefficients by Rivière et al. (2014) improves the correlation accuracy.

Krishnappa & Seetharamiah (1963) also conducted an experimental campaign for bifurcations in the transcritical regime with different branch-width ratios and proposed an alternative discharge distribution correlation. However, when tested against the data of Rivière et al. (2014) (in equal-width configurations), the agreement was unsatisfactory. Rivière et al. (2014) reported that this may be attributed to the inclined side branch used in the author's laboratory configuration. The correlation was then evaluated based on the present experiments, which confirmed the discrepancy with the measured discharge distributions. Similarly, the model put forward by Ramamurthy & Satish (1988) to predict the discharge distribution in the transcritical regime was not considered here, which is consistent with the issues raised by Rivière et al. (2014). Later, Ramamurthy et al. (1990) proposed a momentum-based theoretical model that was tested using laboratory data for transcritical flow. However, the campaign, and the one suggested by Ramamurthy et al. (2007), did not include measurements of water depths in the side branch; thus, these works are not utilized here.

It is noted that at this stage of the present research, no literature has been found on investigations of drowned recirculation in the transcritical regime as described next in Chapter 3.

2.6 SUMMARY OF LITERATURE REVIEW

Based on review of literature, the Bulle Effect is still being researched since its advent in 1926. Even the complex 3D flow patterns and hydrodynamics alone (without considering sediments) in the branch or lateral channel are yet to be fully understood, most likely due to the physical limitations of traditional laboratory experiments. With technological advancement in recent decades, the usage of computer models, especially in computational fluid dynamics (CFD) of late, has provided an opportunity for more detailed numerical investigation of the phenomenon in bifurcating open channel flows. The work of Dutta (2017) is excellent as it explored the wide range of CFD techniques presently available in simulating turbulent flows based on the Navier-Stokes equation. It also included a good start to the modelling of sediments with Lagrangian tracking of particles, and successfully conducted high-resolution numerical simulations for model configurations similar to the experiments by Bulle. This came with a steep price in utilizing super-computing facilities to execute Direct Numerical Simulations (DNS) and

high-resolution Large Eddy Simulations (LES) runs. The key finding is that the relatively less ‘demanding’ Reynolds Averaged Navier-Stokes (RANS)-based numerical models are valid, with further improvement, for simulating flows in bifurcations or diversions.

In specific reference to the Bulle Effect, Momplot et al. (2017) addresses the boundary of flow structures between 2D closed and 3D helix-shaped recirculation patterns – the only study thus far to recognize existence of the latter. It is also part of a body work initiated some years ago by Rivière et al. (2007), and as such utilizes the same experimental setup (with suitable flow aspect ratios) and empirical closure for discharge ratio for dividing flows of equal widths. A total of 16 CFD model (RANS solution in ANSYS Fluent) runs were successfully implemented, validated only by ONE set of experimental results, resulting in Figure 2.9 and the embedded equation. It is then logical to continue such research by conducting substantially more experimental runs to study the effect of varying both channel widths in defining and deriving flow typology as well as distribution.

Based on the summary of key points and parameters by previous investigations provided in Table 2.2, it can be concluded here [similar to Momplot et al. (2017)] that much work is required for cases where the lateral/branch widths (or width ratios) are reduced significantly as compared to the main channel. Thus, the possible research questions to be addressed in the present work for flow in horizontal T-bifurcated rectangular open channels are:

- Identification of 2D closed and 3D helical flow structures or recirculation patterns
- Characterization of the two recirculation types based on flow typology (namely, Froude number)
- Empirical formulation of discharge distribution between main and lateral/branch channels in the subcritical and transcritical flow regimes

Furthermore, from the empirical correlations of relevant past research shortlisted in Table 2.3, all equations thus far depend mainly on Froude number of the flow in the main and/or lateral/branch channels. As this parameter is not directly available and requires measurements of depths and velocities (discharges), recursive or

iterative methods have been introduced by the respective authors to solve for R_q . Thus, there is evidently a need to further address as well as improve these formulations for more robust real-world applications, especially by scientists and engineers out in the field.



Table 2.2 Summary of literature review

Authors/Researchers	Width ratio	Aspect ratio	Experimental, field or computational approach	Data available	Empirical Q distribution	Flow structures	Visualization technique
Band et al. (2020)	1	< 1	Computational - CFD and AI ML	-	-	-	-
Hsu et al., (2002)	1	< 1	Experimental	✓	maximum	Location of contracted region	Dye for location of contracted region
Krishnappa & Seetharamiah (1963)	¼ - 1	<1	Experimental		✓	-	-
Kubit & Ettema (2001)	0.407						
Lakshmana Rao & Sridharan (1967)	¼ - 1	< 1	Experimental with transcritical conditions in branch channel	✓	✓	Separation zone	Observations
Law (1965)	1	< 1	Experimental (subcritical to supercritical)	✓	-	✓	Spraying of chalk powder
Li & Zeng (2009)	1	< 1	Computational - CFD validated with experiments by Hsu et. al. (2002)	-	-	✓	-
Li et al. (2020)	1	< 1	2D computational	✓			
Mignot et al. (2013)	1	< 1	Experimental and computational (CFD)	✓	-	Velocity fields & streamlines	CFD results

Table 2.2 Summary of literature review (continued)

Authors/Researchers	Width ratio	Aspect ratio	Experimental, field or computational approach	Data available	Empirical Q distribution	Flow structures	Visualization technique
Mignot et al. (2014)	1	< 1	Experimental	✓	-	Mixing layer	Injected white dye
Momplot et al. (2013)	1	< 1	Computational - CFD validated by experiments	✓	-	Velocity fields & streamlines	-
Momplot (2014)	1	< 1	Computational - CFD validated by experiments	✓	-	✓	Injected ink
Momplot et al. (2017)	1	< 1	Computational - CFD validated by experiments	✓	-	✓	Injected ink
Neary & Odgaard (1993)	~0.5	< 1	Experimental	-	-	Velocity profiles	-
Neary & Sotiropoulos (1996)	1	> 1	Computational – 3D laminar	-	-	✓	-
Neary et al. (1999)	1	> 1	Computational - CFD validated by experiments	-	-	✓	-
Omidbeigi et al. (2009)	0.4	< 1	Experimental and computational (CFD)	-	-	-	-
Qu (2005)	1	< 1	Experimental	✓	-	Velocity vector plots	-
Ramamurthy & Satish (1988)	1	> 1	Experimental (subcritical to supercritical)	-	Analytical	-	-

Table 2.2 Summary of literature review (continued)

Authors/Researchers	Width ratio	Aspect ratio	Experimental, field or computational approach	Data available	Empirical Q distribution	Flow structures	Visualization technique
Ramamurthy et al. (2007)	1	< 1	Experimental and computational (CFD)	-	-	Velocity vector plots	-
Rashwan & Saafan (2004)	1	< 1	Experiments by Hsu et. al. (2002)	✓	Analytical	-	-
Rivière et al. (2007)	1	< 1	Experimental	✓	✓	✓	Observations
Rivière et al. (2011)	1	< 1	Experimental	✓	✓	✓	Observations
Rivière et al. (2014)	1	< 1	Experimental with transcritical conditions in branch channel	✓	✓	✓	Observations
Sayed (2019)	½, ¾, 1	< 1	Experimental (subcritical to supercritical)	✓	✓	-	-
Shaban Eissa et al. (2004)	1	< 1	Experimental with transcritical conditions in both channels	-	-	Velocity vector plots	-
Shettar & Murthy (1996)	1	< 1	Computational – 2D validated by experiments	-	-	Streamlines	-
Weber et al. (2001)	1	< 1	Experimental	✓	-	Velocity vector plots	-

Table 2.3 Summary of discharge distribution investigations and correlations

Authors/Researchers	Flow regime in lateral/branch channel		Equation(s)	Remarks
	subcritical	transcritical		
Hager (1991), Hager (2010)	✓		$R_q = 1 - \tanh[5(A - F_{yo})F_{xo}^{1/2}]$ equation (4.6), where $A = 0.56$ for $R_b = 0.5$	based on measurements by Lakshmana Rao et al. (1968), only for $R_b = 0.5$ without any impact of upstream Froude number (F_{bx})
Hsu et al. (2002)	✓		$\max Q_{yo} = \{1 - \{F_{xo}^2 / [(2 + F_{xo}^2) / 3]^3\}^{1/2}\}$	based on momentum balance approach, with a contraction coefficient
Krishnappa & Seetharamiah (1963)		✓	$R_q(1.545 - 1.45F_{bx})R_b(1 - 2F_{bx})$	$0.25 \leq R_b \leq 1$; $0.2 \leq F_{bx} \leq 0.8$; the agreement was unsatisfactory when tested against the data of Rivière et al. (2014) ($R_b = 1$)
Lakshmana Rao & Sridharan (1967)		✓	$\log_{10}(R_q) + KF_{xo}^\alpha = 0$ equation (4.9)	90° bifurcation; $R_b = 0.25$, $K = 1.4$ and $\alpha = 0.672$; $R_b = 0.5$, $K = 1.27$ and $\alpha = 0.86$; $R_b = 0.75$, $K = 1.16$ and $\alpha = 0.938$; $R_b = 1$, $K = 1.11$ and $\alpha = 0.995$
Lakshmana Rao et al. (1968)	✓		no equation provided, function based on dimensional analysis: $R_q = f(\theta, R_b, F_{xo}, F_{yo})$	only $R_b = 0.5$ was tested

Table 2.3 Summary of discharge distribution investigations and correlations (continued)

Authors/Researchers	Flow regime in lateral/branch channel		Equation(s)	Remarks
	subcritical	transcritical		
Ramamurthy & Satish (1988)		✓	$R_q = R_b \frac{C_c}{\eta_1}$	theoretical model based on momentum; dependent on contraction coefficient C_c and velocity parameter η_1 ; proposed graphs for practical applications
Ramamurthy et al. (1990)		✓	-	no depth measurements provided
Ramamurthy et al. (2007)		✓	-	no depth measurements provided
Rashwan & Saafan (2004)		✓	$\frac{Q_{xo}}{Q_{xi}} = \sqrt{\frac{h_x/h_{xo} (2 + F_{bx}^2) - 2}{F_{bx}^2 (h_x/h_{xo})^3}}$	validated using Hsu et al. (2002) data (subcritical regime)

Table 2.3 Summary of discharge distribution investigations and correlations (continued)

Authors/Researchers	Flow regime in lateral/branch channel		Equation(s)	Remarks
	subcritical	transcritical		
Rivière et al. (2007)	✓		$R_q = 0.5 - 0.04675F_{Cx} + 0.358 \left(1 - \frac{C_y}{C_x}\right) F_{Cx}^{-0.77}$ equation (4.7)	restricted to $R_b = 1$ and neglecting the effect of C_x/b_x ; not adapted to field conditions
Rivière et al. (2011)	✓		$R_q = [1 - A - B + AR_c]R_{qi} + B + D(R_c - 1)(R_b + E)(R_g^{-1} + F)$	$A = -0.141, B = 0.456, D = -0.113,$ $E = -0.030, F = 9.797$ developed for 4-branch intersections, where $R_{qi} = 0$ for T-bifurcations and $R_g = Q_{xi}/(b^2\sqrt{2g})$
Rivière et al. (2014)		✓	$\log_{10}(R_q) + 0.925F_{xo}^{0.98} = 0$	90° bifurcation, $R_b = 1$; adjusted coefficients in equation (4.9) of Lakshmana Rao & Sridharan (1967) to $K = 0.925$ and $\alpha = 0.98$

CHAPTER THREE

STUDY METHODOLOGY

3.1 INTRODUCTION

As described in Chapter 1 (Introduction) and reviewed in Chapter 2 (Literature Review), recent investigations of open-channel bifurcations, natural or otherwise, are concerned mainly with flow division, in terms of behaviour and quantification, within the main and branch waterways. The primary reference is the work initiated by Vincent S. Neary in his 1992 MSc study, “Flow Structure at an Open Channel Diversion”, at the University of Iowa.

Based on experimental results, Neary and Odgaard (1999) concluded that diversion flow is distinctly three dimensional and more readily diverted on channel bed/bottom. In addition, a disproportionate amount of bed sediments would enter diversion as compared to main channel. Such complex three-dimensional (3D) flow separation and patterns were also obtained from numerical solutions for steady, laminar (Neary and Sotiropoulos, 1996) and turbulent (Neary and Odgaard, 1999) conditions in rectangular open-channel T-junctions. It is noted here that the “closed semi-elliptic region developing along the upstream wall of the lateral branch” is inferred to be 2-dimensional.

Momplot et al. (2017) separated the flow behaviour in a right-angle diversion channel into 2D closed and 3D helix-shaped recirculation patterns which are distinct from each other. Experimental tests and computational simulations indicate that these are dependent on and can be estimated by Froude number and the upstream or main channel aspect (flow depth to width) ratio.

In the study methodology for this research, experiments have been conducted firstly to further examine the computational simulations and results published by Momplot et al. (2017). Based on the preceding work of Rivière et al. (2007) and Mignot et al. (2013) the same laboratory setup established at Laboratoire de Mécanique des

Fluides et d'Acoustique (LMFA) of Institut National des Sciences Appliquées (INSA) in Lyon, France have been utilized after some reorganization of the configuration, for this purpose. A series of flume runs was then initiated with the original equal-width channels to validate findings of the previous study. The plan was to complete these first, followed by repeating the runs for the narrower branch width cases and conduct further rigorous investigation of bifurcated flow. Originally, all experiments were scheduled over a three-to-four-month period beginning late January 2020, but due to the Covid-19 pandemic crisis and ensuing lockdowns, only the first set of experiments have been accomplished in March 2020. The second part was then executed later in middle 2021.

3.2 AIM OF EXPERIMENTAL STUDY

In addressing objectives 1 and 2 stipulated in Chapter 1 (Introduction) for this research, the experiments are designed to achieve the following targets:

- to investigate the findings of Momplot et al (2017) with actual measurements in the flume at LMFA, INSA Lyon,
- to identify the two distinct flow structures, i.e. 2D standard closed recirculation and complex 3D helical patterns, utilizing tracer ink or other reliable, consistent technique, and
- to derive new discharge distribution formulations of bifurcated flows based on a series of actual measurements in the flume.

The investigative process and procedures implemented at laboratory facilities of LMFA, INSA Lyon is summarized in the following flow-chart (Figure 3.1):

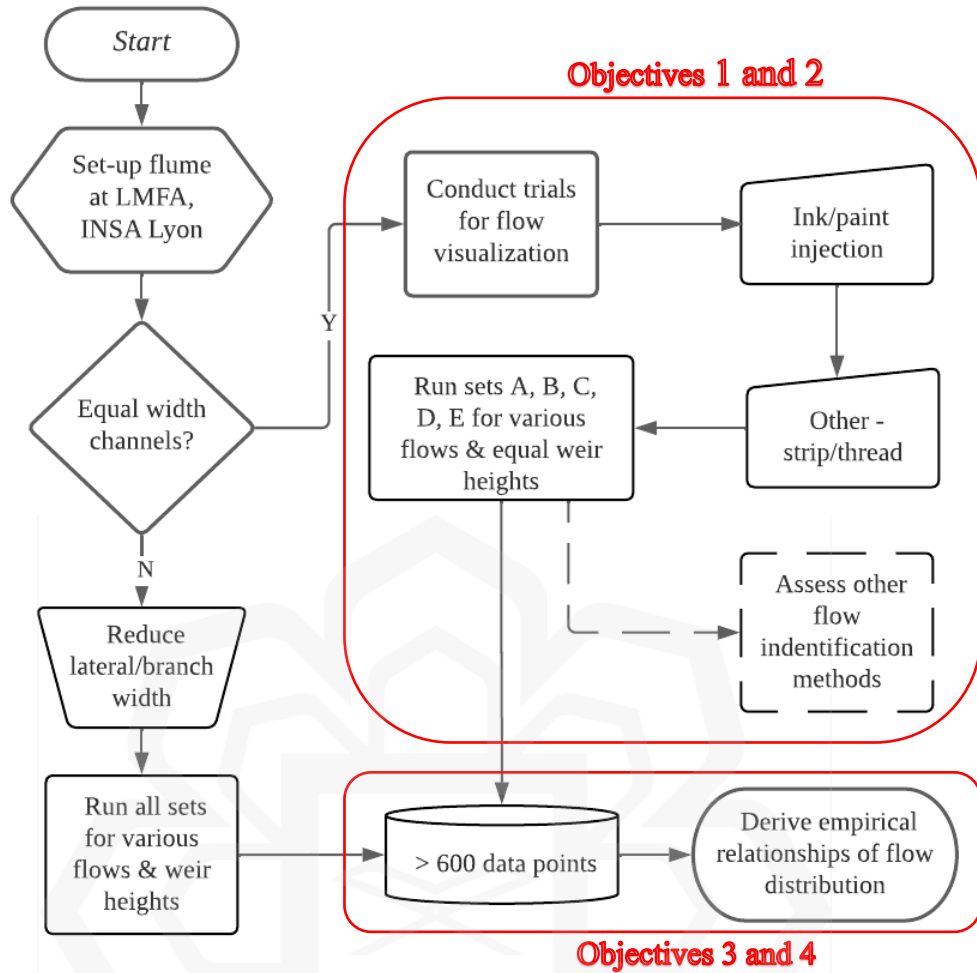


Figure 3.1 Flowchart of experimental study

3.3 LABORATORY AND FLUME SETUP

The whole experimental setup and configuration consisting of open-channel flume (in an enclosed box), tanks, pump, hoses, connections, valves, depth gauges, is portrayed schematically in Figure 3.2. As stated above, the layout of Figure 3.2(a) has been rearranged from the endeavours of Rivière et al. (2007), Mignot et al. (2013) and Momplot et al. (2017), with some adjustments and improvements for extended test re-runs and further examination of flow patterns.

3.3.1 Equal channel width

In the first set of experiments, widths of the main and lateral/branch channels have been kept equal based on the then existing setup of previous investigations. A more detailed description of the systems is given in Rivière et al. (2011) – although it was mainly established for studying four-branch intersecting open channel flow, the arrangement is easily adapted to the present three-branch analysis. It is noted here that while the channel are all equally wide, i.e. b_x and b_y are both 30 cm, the system has been improved to also accommodate reduced lateral/branch widths, as depicted in Figure 3.2(b), to address the next phase of experiments and analysis. The following summarizes the experimental set-up as well as material and equipment used (Rivière et al., 2011):

- each channel is made of glass with dimensions (main channel is 4.9 m long whilst the lateral or branch is 2.6 m long) indicated in Figure 3.2
- a portion of the channels are enclosed ('Box' in Figure 3.2) to maintain minimal light conditions when required (such as for photo and video capture)
- x and y denote the main and lateral/branch directions, respectively
- both main and lateral/branch channels are horizontal (0% slope which can be adjusted independently)
- adjustable sharp-crested weirs are installed at the outlets of main and lateral/branch channels with crest heights, C_x and C_y , respectively, to control water depth
- in this closed flow system, controlled and regulated by a set of valves, water is pumped from the downstream main tank to the upstream chamber
- a polyurethane float in the upstream tank reduces surface waves as flow is directed to the main channel via a stabilizing honeycomb screen
- inlet flow rate Q_{in} varies between 0 and 12 l/s
- discharges at the inlet and the outlets are measured by three identical Endress & Hauser Promag 50 electromagnetic flowmeters with ± 0.05 l/s accuracy

- water depths are measured using a mobile point gauge with ± 0.15 mm uncertainty (no surface waves) [ranging between 4 and 10 cm, with a typical 0.5 mm uncertainty]
- in addition to the 2011 system, flows and depths were also monitored by a computerized system with National Instruments' LabVIEW 7 Express during the 2020 phase (this study)



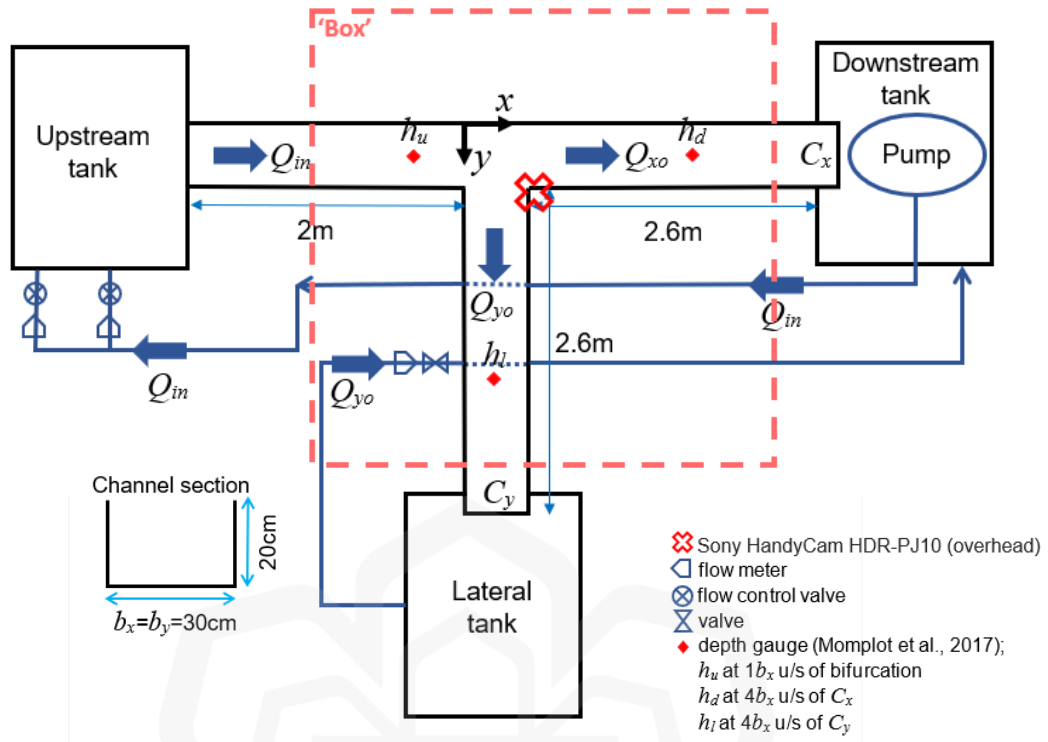


Figure 3.2(a) Experimental setup of Momplot et al. (2017)

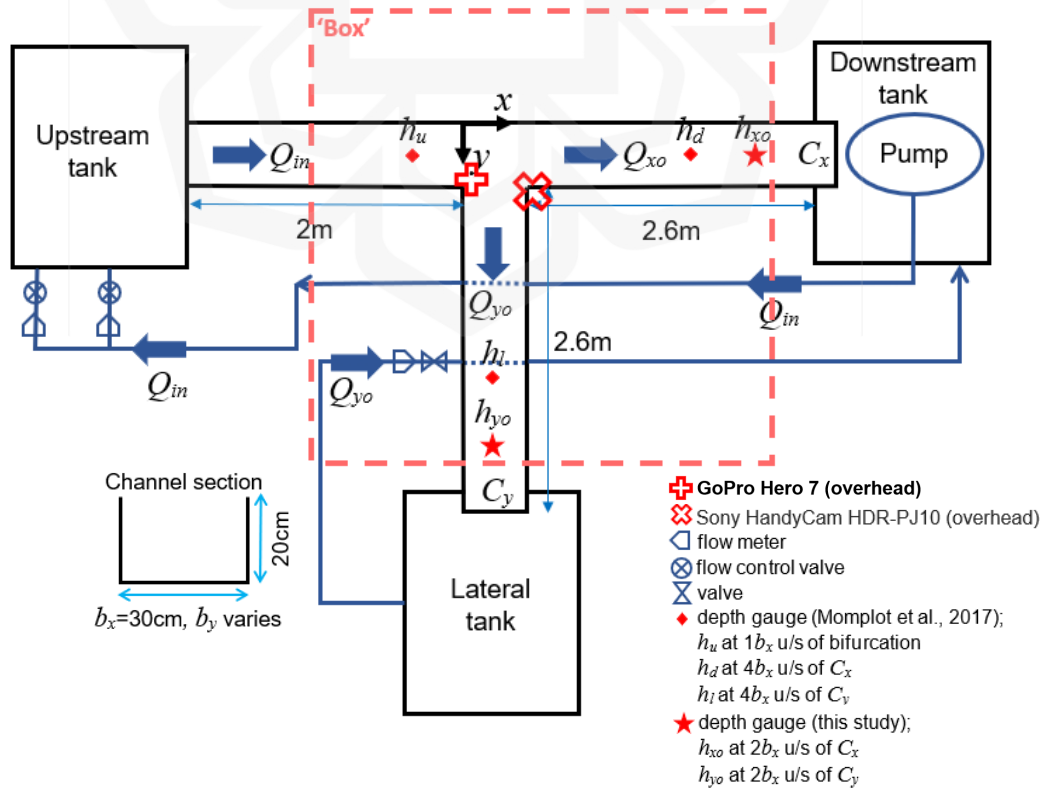


Figure 3.2(b) Experimental setup updated for this study



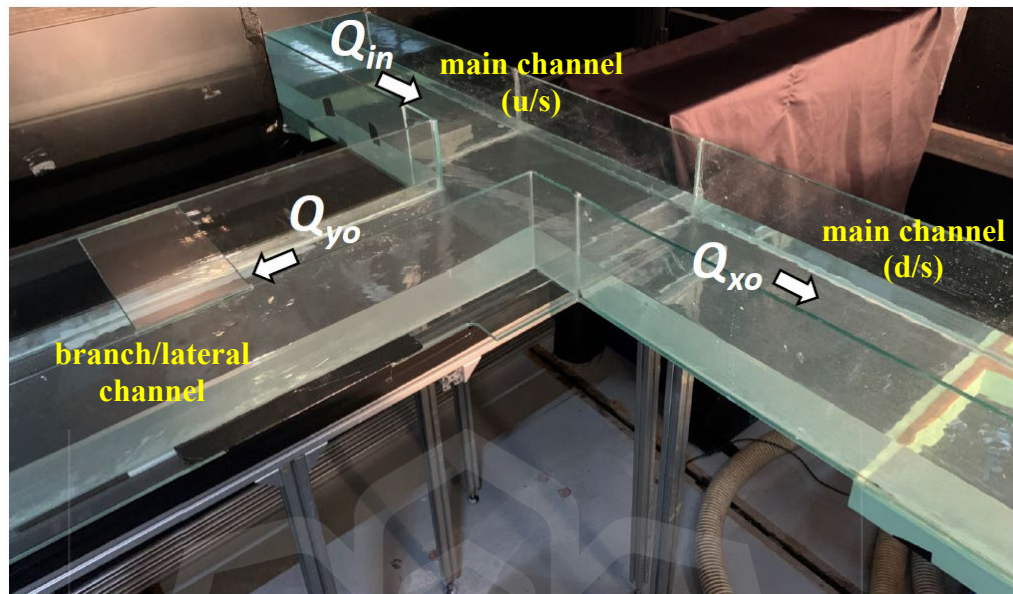


Figure 3.3a Photos of experimental setup – main and lateral/branch channels

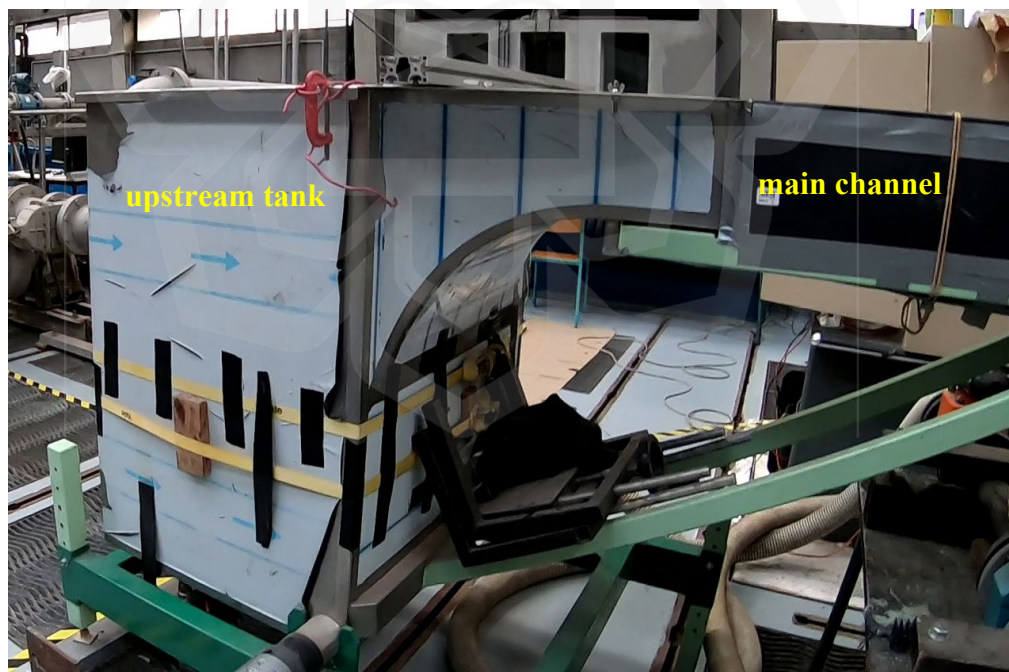


Figure 3.3b Photos of experimental setup – upstream tank and main channel

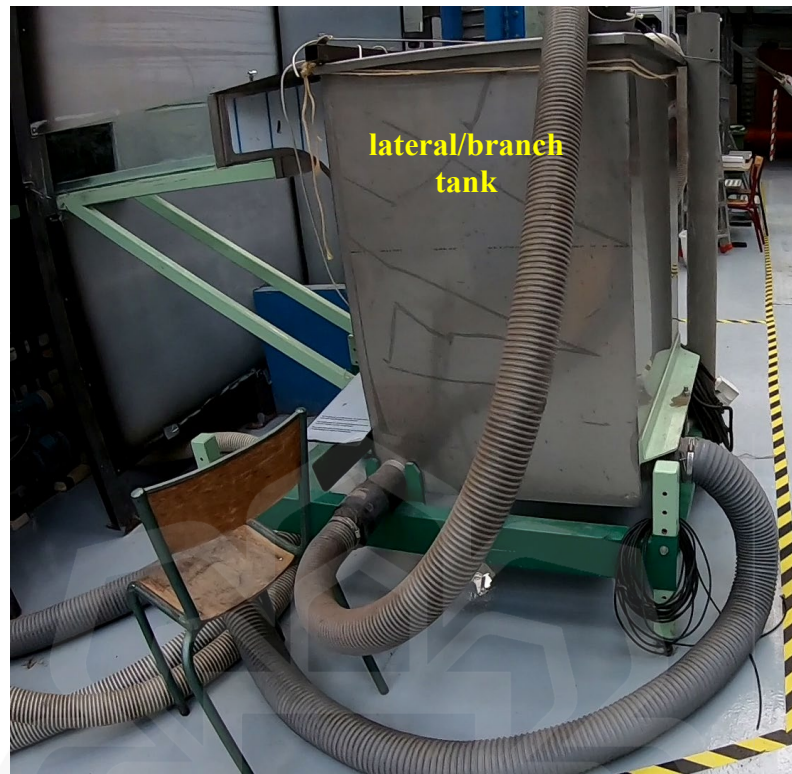


Figure 3.3c Photos of experimental setup – lateral/branch tank and weir



Figure 3.3d Photos of experimental setup – downstream tank, weir and submersible pump

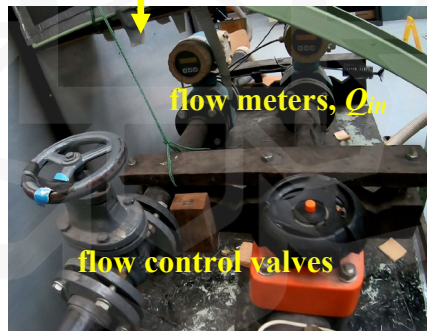
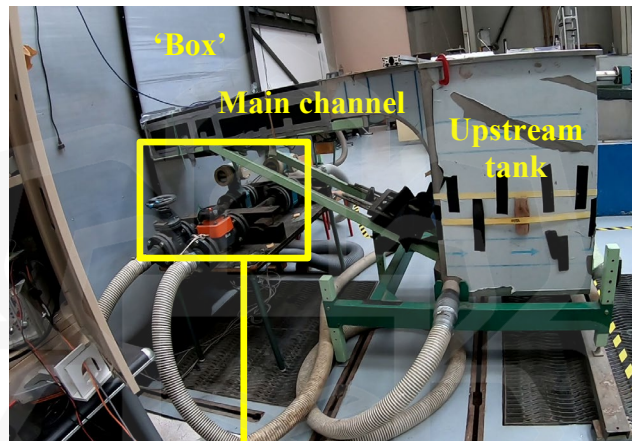
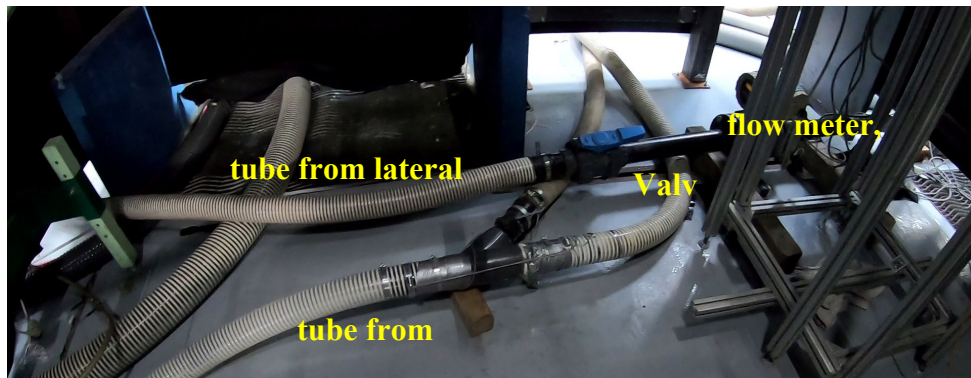


Figure 3.3e Photos of experimental setup – flow meters, valves, connections, etc.



Figure 3.3f Photos of experimental setup – computerized monitoring with National Instruments' LabVIEW 7 Express

3.3.2 Reduced Lateral or Branch Width

In the first phase, the experiments have been conducted in the same flume as in test runs by Rivière et al. (2007), Mignot et al. (2013) and Momplot et al. (2017) with very limited modifications. Based on this system consisting of two 0.3 m wide rectangular shape horizontal and smooth channels made of glass, intersecting at right angle at about mid-length of the main branch [Figure 3.2(a)], the next stage of investigation proceeded with width reduction of the lateral/branch channel as indicated by Figure 3.2(b). This was accomplished by adding a wall parallel to the side walls in the lateral/branch and a second short wall in the continuity of right main channel wall to enclose this reduced area (Figure 3.4).

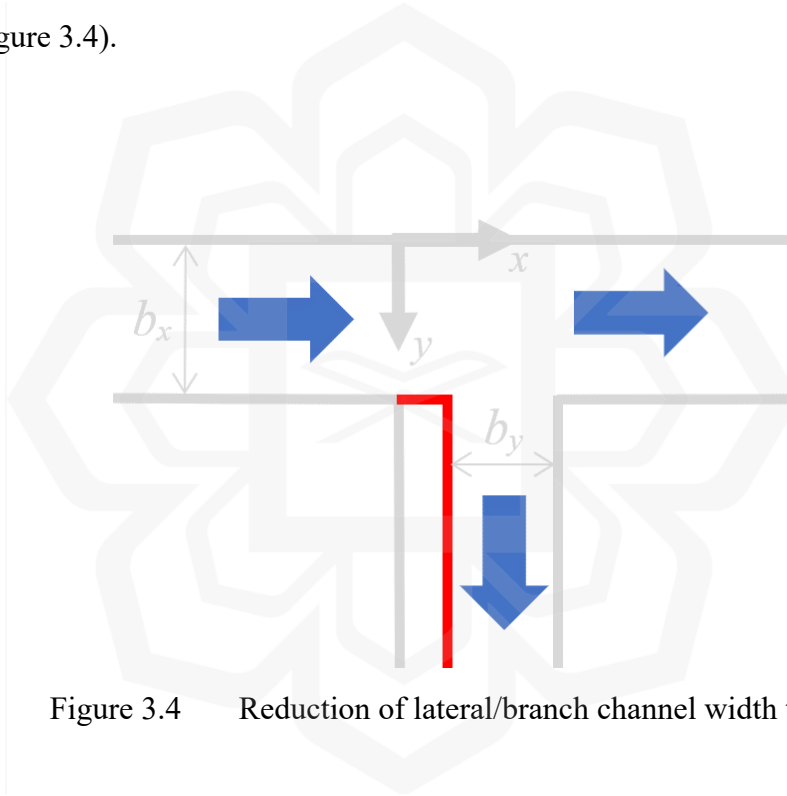


Figure 3.4 Reduction of lateral/branch channel width to b_y

Including the first set of runs, five lateral/branch width ratios are considered herein with $R_b = b_y/b_x = 1, 26/30, 22/30, 18/30$ and $10/30$.

3.4 DIMENSIONAL ANALYSIS AND DESIGN OF EXPERIMENTS

3.4.1 Dimensional Analysis

Based on the channel configuration and geometry presented in Figure 3.2 with flow under steady conditions, the value of the discharge reaching the side branch Q_{yo} (or alternatively reaching the downstream branch Q_{xo}) is dependent on 12 input dimensional variables defined as follows:

$$Q_{yo} = f(g, \rho, \mu, \sigma, k_s, Q_{in}, h_{xo}, h_{yo}, C_x, C_y, b_x, b_y) \quad (3.1)$$

with g the gravity, ρ and μ the water density and dynamics viscosity, σ the air/water surface tension and k_s the equivalent roughness elevation. By applying the Vaschy-Buckingham theorem, the non-dimensional discharge distribution correlation as a function of 9 dimensionless parameters is formulated as:

$$R_q = \frac{Q_{yo}}{Q_{in}} = f \left(Re_{bx} = \rho \frac{Q_{in}}{b_x \mu}, We = \rho \frac{Q_{in}^2}{b h_{xo}^2 \sigma}, \frac{k_s}{b_x}, F_{bx} = \frac{Q_{in}}{b_x^{5/2} g^{1/2}}, \frac{h_{xo}}{b_x}, \frac{h_{yo}}{b_x}, \frac{C_x}{b_x}, \frac{C_y}{b_x}, R_b = \frac{b_y}{b_x} \right) \quad (3.2)$$

However, considering the low roughness thickness of glass material as well as significantly high velocities and water depths, the flow remains in hydraulically smooth and fully turbulent regimes, with negligible effects of surface tension, so that Eq. (3.2) reduces to:

$$R_q = f \left(F_{bx}, \frac{h_{xo}}{b_x}, \frac{h_{yo}}{b_x}, \frac{C_x}{b_x}, \frac{C_y}{b_x}, R_b \right) \quad (3.3)$$

Considering the stage-discharge relationship at both weirs [$Q_o = f(C_{x,y}, h_o)$], either respective depth or height parameter in Eq. (3.3) can be excluded. Among possible combinations of dimensionless parameters, two versions of this function read as follows:

$$R_q = f \left(F_{Cx} = \frac{Q_{xi}}{b_x g^{1/2} C_x^{3/2}}, \frac{C_x}{b_x}, \frac{C_y}{C_x}, R_b \right) \quad (3.4)$$

$$R_q = f \left(F_{bx}, F_{xo} = \frac{Q_{xo}}{b_x g^{1/2} h_{xo}^{3/2}}, F_{yo} = \frac{Q_{yo}}{b_y g^{1/2} h_{yo}^{3/2}}, R_b \right) \quad (3.5)$$

In equation (3.4), following Rivière et al. (2007) for a T-bifurcation or Rivière et al. (2011) for a 4-branch intersection, the dimensionless parameters are based on weir crest heights. Alternatively, equation (3.5) is based on water depths, which is valid regardless of the type of downstream control (weir, dam, lake, free-fall, etc.) that links the local water depth and discharge.

In addition, as highlighted in Chapter 2, with the main flow bifurcating and entering the lateral/ branch channel, Momplot et al. (2017) identified a balance of the centrifugal and volume pressure forces that generates either 2D closed or 3D helical patterns in the recirculation zone. This was progressively reduced to the fraction of the squared upstream Froude no. to upstream aspect ratio, i.e. $(F_{bx})^2$ over b_x/h_u , which completely defines “the flow patterns and establish a typology of the flow structure in the lateral branch”.

3.4.2 Design of Experiments

To establish a sound design for the experiments, initially the runs were planned in accordance with the work of Momplot et al. (2017) (both for equal-width channels) as presented in Table 3.1 and Figure 3.5. Based on their finding, the linear equation 6 of Momplot et al. (2017) as exemplified by Figure 2.9, is also plotted in Figure 3.5.

The points plotted on the straight vertical lines representing equal b_x/h_u require that upstream flow depths, h_u , were to be kept constant over various total (upstream) discharges – ideally, this is achieved by adjusting both lateral and downstream weir heights ($C_x = C_y$) while fixing Q_{in} . However, with arduous fine tuning of the weirs required to achieve a certain flow rate, it is difficult to physically execute and sustain

such conditions in the flume – note that this could be easily implemented with computational fluid dynamics (CFD simulations, as is the case in Momplot et al. (2017).

Table 3.1 Initial design of experimental runs based on Momplot et al. (2017)

Momplot et al (2017)									experimental parameters to be fixed			
Run	case	b_x	$C_x = C_y$	h_u	b_x/h_u	Q_{in}	Q_{yo}/Q_{in}	$(F_{bx})^2$	h_u	b_x/h_u	$(F_{bx})^2$	Q_{in}
		m	m	m		m ³ /s			m			l/s
2	F2	0.3	0.03	0.048	6.20	0.004	0.45	0.160	0.048	6.20	0.15	3.87
									0.048	6.20	0.18	4.24
									0.048	6.20	0.19	4.36
									0.048	6.20	0.20	4.47
									0.048	6.20	0.22	4.69
									0.048	6.20	0.25	5.00
7		0.3	0.05	0.078	3.84	0.008	0.457	0.152	0.078	3.84	0.05	4.59
									0.078	3.84	0.06	5.03
									0.078	3.84	0.08	5.80
									0.078	3.84	0.09	6.16
									0.078	3.84	0.10	6.49
									0.078	3.84	0.15	7.95
15	F1	0.3	0.09	0.120	2.50	0.008	0.468	0.042	0.120	2.50	0.01	3.91
		0.3	0.09	0.120	2.50				0.120	2.50	0.02	5.52
									0.120	2.50	0.03	6.77
									0.120	2.50	0.04	7.81
									0.120	2.50	0.05	8.73
									0.120	2.50	0.06	9.57
16	F0	0.3	0.09	0.150	2.00	0.012	0.474	0.048				
		0.3	0.098	0.120	2.50	0.004	0.51	0.010				

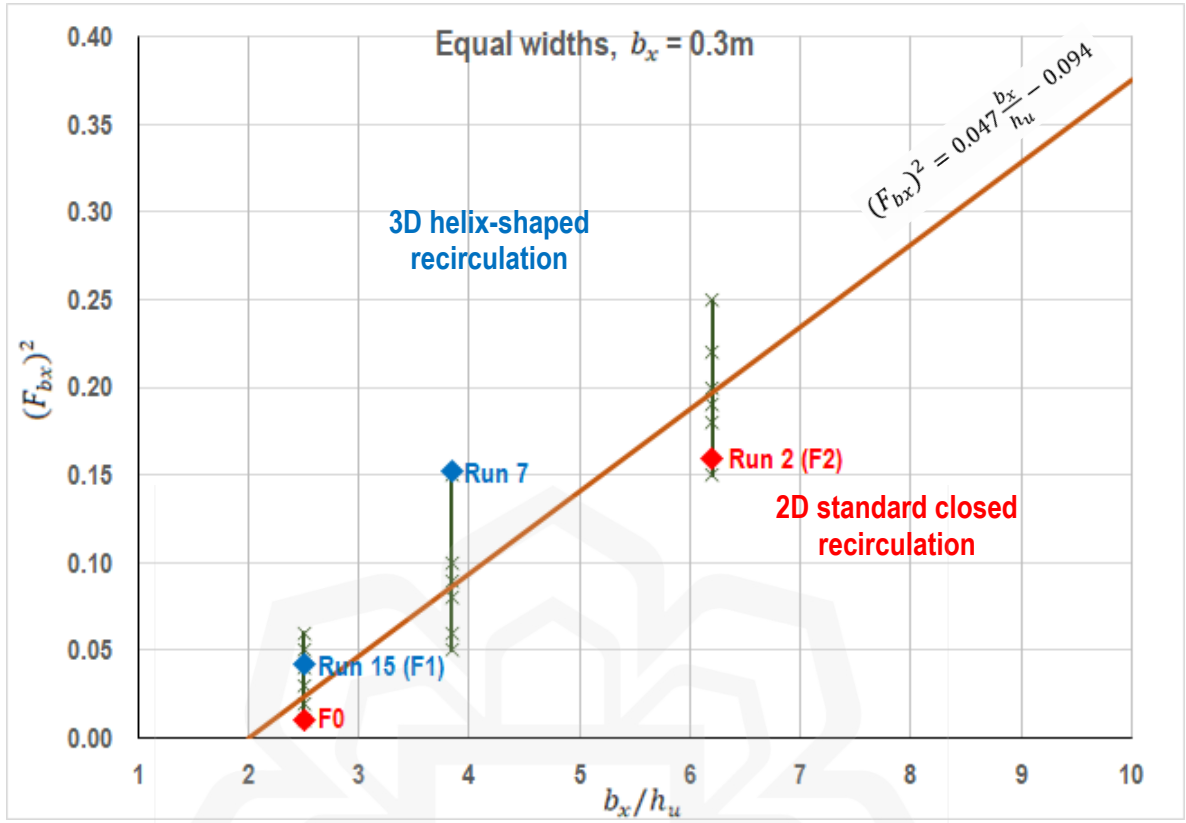


Figure 3.5 Initial design of experimental runs based on Momplot et al. (2017)

Therefore, the experiments were modified for practical purposes by first fixing the weirs at equal heights of $C_x = C_y$. The flows in the closed system, starting from the pump in the downstream storage that fed into the inlet tank, were then regulated over a typical range of 2 to 10 l/s by adjusting the upstream control valves (Figure 3.2 and Figure 3.3e). Once the conditions had stabilized, i.e. when the flowmeters did not exhibit significant fluctuations in readings for a certain discharge Q_{in} , flow depth h_u in the main channel was measured at one width ($1b_x$) distance upstream from the intersection. As discussed later in Chapter 4, the resulting $(F_{bx})^2$ versus b_x/h_u data points would be plotted as curves rather than the straight vertical lines of Figure 3.5. After some testing with trial runs, it was deemed that this procedure was practical and feasible, thus the design of equal-width runs to address experimental targets 1 and 2, i.e. to investigate the findings of Momplot et al. (2017) and distinguish the two distinct flow

structures, is established for a series of weir heights and inlet flows as summarized in Table 3.2. In reference to Figure 3.2, the measurement variables are:

- h – flow depth above weir (Rehbock, 1929)
- h_d – downstream flow depth in main channel [Momplot et al. (2017)]
- h_l – flow depth in branch channel [Momplot et al. (2017)]
- h_u – upstream flow depth in main channel
- h_{xo} – downstream flow depth in main channel
- h_{yo} – flow depth in lateral/branch channel
- Q_l – discharge in lateral/branch channel [Momplot et al. (2017)]
- Q_{yo} – discharge in lateral/branch channel

To derive new discharge distribution formulations of bifurcated flows (third target), additional runs are required for reduced lateral/branch widths, namely $R_b = b_y/b_x = 26/30, 22/30, 18/30$ and $10/30$. These are listed in Table 3.3 for equal weir heights, as well as in Appendix A for all equal and variable weir heights. A total of 668 runs have been planned and executed for this study.

Table 3.2 Completed experimental runs for equal-width channels ($C_x = C_y$)

$C_x, C_y = 3 \text{ cm}$		$C_x, C_y = 4 \text{ cm}$		$C_x, C_y = 5 \text{ cm}$		$C_x, C_y = 6 \text{ cm}$		$C_x, C_y = 7 \text{ cm}$		$C_x, C_y = 9 \text{ cm}$	
Run	$Q_{xi} (l/s)$	Run	$Q_{xi} (l/s)$	Run	$Q_{xi} (l/s)$	Run	$Q_{xi} (l/s)$	Run	$Q_{xi} (l/s)$	Run	$Q_{xi} (l/s)$
A1	2.42	B1	1.92	C1	2.76	D1	2.35	E1	2.31	F1	2.42
A2	3.56	B2	2.6	C2	3.55	D2	3.3	E2	3.12	F2	2.88
A3	3.86	B3	3.19	C3	4.22	D3	4.08	E3	3.91	F3	3.38
A4	4.27	B4	3.65	C4	4.95	D4	4.73	E4	4.77	F4	4.01
A5	5.06	B5	4.19	C5	5.5	D5	5.31	E5	5.622	F5	4.6
A6	5.55	B6	4.77	C6	6.17	D6	6	E6	6.17	F6	5.08
A7	6.03	B7	5.2	C7	6.9	D7	6.52	E7	6.81	F7	5.55
A8	6.67	B8	5.86	C8	7.5	D8	6.95	E8	7.48	F8	6
A9	7.24	B9	6.38	C9	8.02	D9	7.46	E9	8.08	F9	6.77
A10	7.81	B10	6.99	C10	8.73	D10	8.02	E10	8.96	F10	7.57
A11	8.2	B11	7.79	-	-	D11	8.48	-	-	-	-
A12	8.94	B12	8.53	-	-	-	-	-	-	-	-
-	-	B13	9.25	-	-	-	-	-	-	-	-

Table 3.3 Completed experimental runs for reduced lateral/branch widths ($C_x = C_y$)

R_b	$Q_{xi}(l/s)$	C_x, C_y (cm)	R_b	$Q_{xi}(l/s)$	C_x, C_y (cm)	R_b	$Q_{xi}(l/s)$	C_x, C_y (cm)	R_b	$Q_{xi}(l/s)$	C_x, C_y (cm)
10/30	2.64	4	18/30	2.58	6	22/30	2.34	4	26/30	2.36	5
	2.87	7		2.69	3		2.52	5		2.49	7
	2.9	5		2.69	7		2.52	7		2.63	4
	2.95	6		2.8	4		2.66	6		2.64	3
	2.99	3		3.08	5		2.77	3		2.8	6
	3.4	4		3.17	6		2.97	4		2.88	5
	3.5	7		3.28	7		3.04	6		3.13	4
	3.64	3		3.42	3		3.05	7		3.13	7
	3.68	5		3.48	4		3.31	5		3.36	5
	3.78	6		3.72	6		3.452	3		3.38	6
	3.99	4		3.85	5		3.52	7		3.64	3
	4.09	7		3.92	7		3.67	4		3.68	7
	4.32	3		4.07	4		4.07	3		3.77	6
	4.36	5		4.16	3		4.08	4		3.81	4
	4.43	6		4.44	6		4.09	7		3.98	5
	4.55	7		4.45	5		4.12	5		4.26	3
	4.73	4		4.53	7		4.38	6		4.35	7
	4.98	5		4.82	4		4.45	4		4.38	6
	5.14	3		4.84	3		4.49	7		4.53	4
	5.16	6		5.04	6		4.59	3		4.54	5
	5.21	7		5.07	5		4.71	5		4.85	3
	5.49	5		5.1	7		5.03	4		4.9	7
	5.7	3		5.44	4		5.13	6		4.91	6
	5.72	6		5.47	3		5.25	3		5.05	4
	5.84	4		5.6	6		5.33	7		5.39	7
	5.94	5		5.73	7		5.41	5		5.47	3
	6.02	7		5.77	5		5.61	4		5.48	5
	6.2	3		6.12	4		5.79	3		5.49	6
	6.26	6		6.21	6		5.83	6		5.73	4
	6.35	4		6.31	3		5.84	5		5.88	7
	6.61	5		6.31	7		6.04	7		5.95	3
	6.64	7		6.47	5		6.08	4		6	6
	6.72	6		6.75	4		6.34	5		6.04	5
	6.83	3		6.93	3		6.46	3		6.25	4
	6.98	4		6.94	6		6.55	6		6.41	7
	7.11	7		6.96	7		6.72	4		6.49	5
	7.14	5		7.17	5		6.78	7		6.59	6
	7.32	6		7.29	4		7.04	3		6.61	3
	7.63	4		7.51	7		7.16	5		6.83	4
	7.66	3		7.52	6		7.2	6		6.89	7
	7.8	5		7.68	3		7.37	7		6.99	5
	7.87	6		7.84	5		7.38	4		7.02	3
	7.95	7		8.11	4		7.64	3		7.12	6
	8.4	3		8.13	7		7.86	5		7.43	4
	8.49	6		8.16	3		7.88	6		7.47	5
	-	-		8.16	6		7.99	7		7.54	3
	-	-		8.39	5		8.45	6		7.54	6
	-	-		8.59	3		8.48	5		7.58	7
	-	-		-	-		-	-		7.92	3
	-	-		-	-		-	-		7.97	4

3.5 IDENTIFICATION OF FLOW STRUCTURES AND PATTERNS

To specifically address the first and second objectives of this research stipulated in Chapter 1 (Introduction), flow visualization techniques were explored to distinguish the two different bifurcated flow structures as reported by Momplot et al. (2017). Based on analysis of computational results generated for several cases (with subcritical conditions in the main channel) by their ANSYS Fluent CFD model, these investigators concluded that flow within the separation zone of the lateral/branch channel exhibited either 2D standard recirculation or 3D helical patterns (Figure 2.8).

It is noted here that while Momplot et al. (2017) manually injected white ink from the downstream end of the main channel at the bifurcation in three experimental runs (one for validation of the CFD model, two for visualization), the process in the present work started with early tests and trials to establish the appropriate tracer material (ink or paint) and insertion method that is practical as well as consistent over the range of experimental discharges. Unfortunately, the previously established manual injection could not be consistently replicated in the flume, due to transients, sporadic conditions as well as relatively high turbulence in the actual flows. To minimize some of these issues, a tank-supply system was installed above the flume to provide consistency in the material insertions. Videos of such tracer performance within the bifurcation, especially in the recirculation zone at the lateral or branch channel's upstream wall, have been captured on the overhead camera (Figure 3.2) for all affected runs. The effectiveness of this method is assessed based on such recordings of the trial runs (alluded to earlier) with the data and results presented in Table 3.4.

Similarly to Figure 3.5, the data of Trial 1, 2, 3, 7 plotted in Figure 3.6 are located below, while Trial 4, 5, 6, 8, 9 above, the linear (equation) line of Momplot et al. (2017). These positions indicate that the former flow set was of standard 2D closed rotational behaviour, while the latter resulted in 3D helix-shaped patterns, in the recirculation zone as identified by the previous investigation.

Table 3.4 Trial experimental runs for equal-width channels (equal weir heights)

measurements			$(F_{bx})^2$ vs b_x/h_u		
Trial	Q_{in} (l/s)	h_u (mm)	b_x/h_u	$(F_{bx})^2$	F_{bx}
$C_x=C_y=30\text{mm}$					
1	1.77	41.6	7.20	0.049	0.22
2	4.46	51.2	5.86	0.168	0.41
3	4.65	51.8	5.79	0.176	0.42
4	5.02	53.0	5.66	0.192	0.44
5	5.19	53.5	5.61	0.200	0.45
6	8.09	60.3	4.98	0.339	0.58
$C_x=C_y=90\text{mm}$					
7	3.9	111.5	2.69	0.012	0.11
8	7.75	124.9	2.40	0.035	0.19
9	9.55	130.2	2.30	0.047	0.22

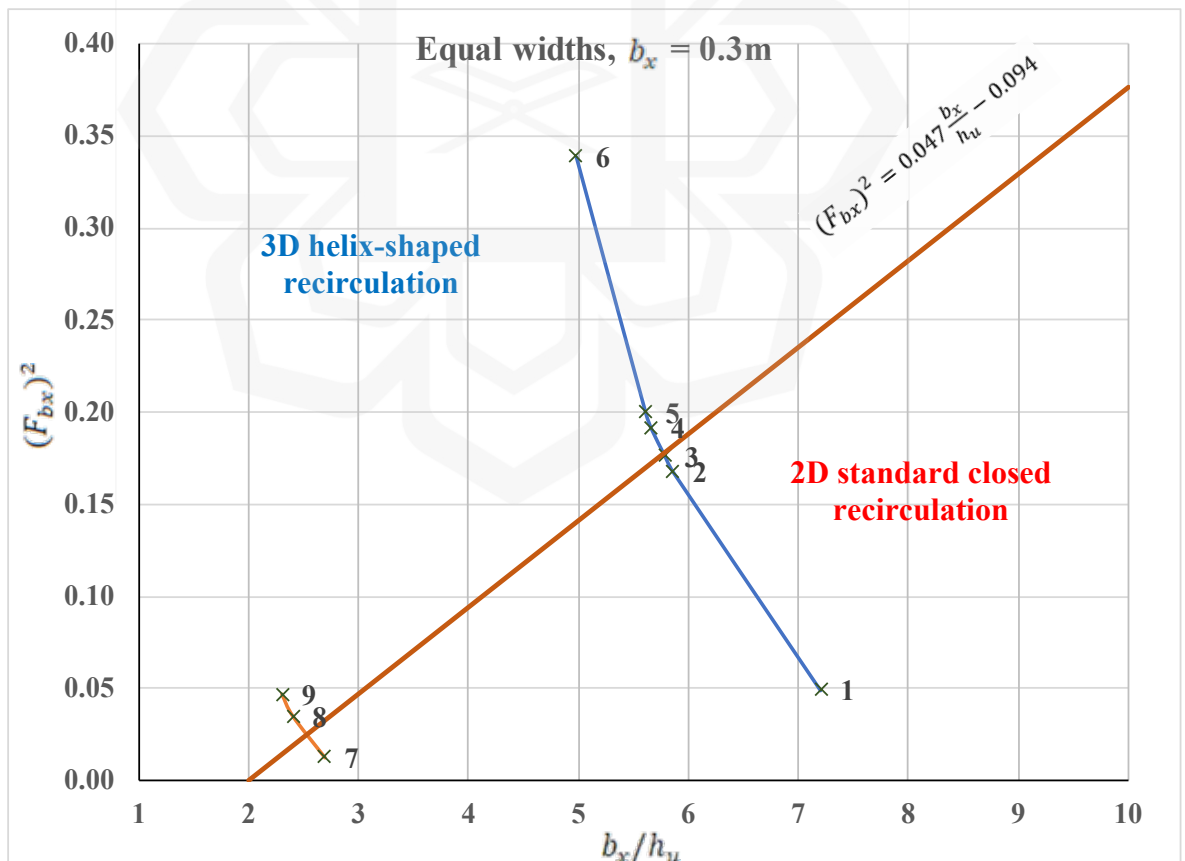
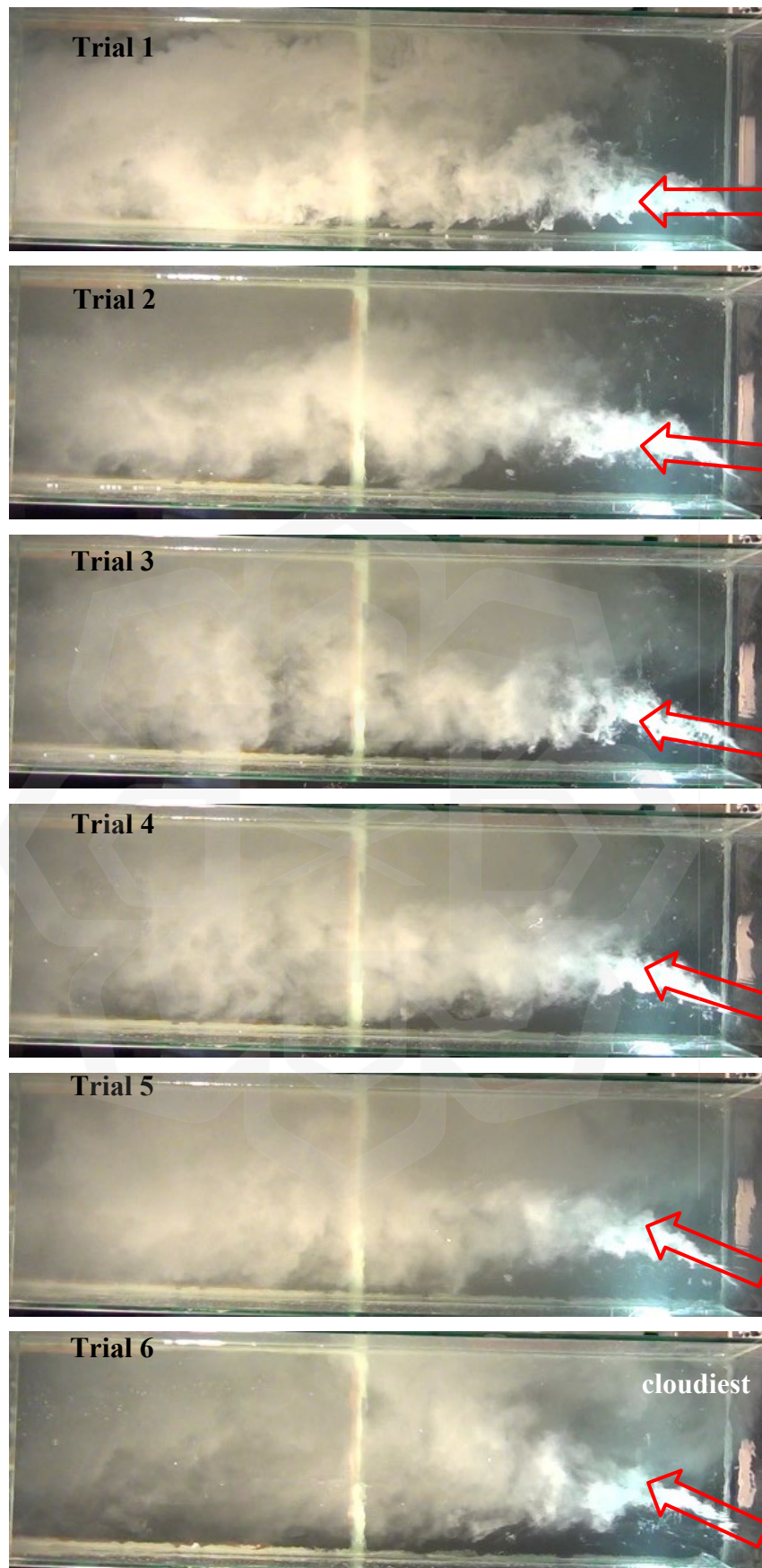


Figure 3.6 Trial 1 to 9 data as compared to linear equation and line developed by Momplot et al. (2017)

Upon close inspection of the videos snapshots for all the trial runs above, it is concluded that modifications with a constant supply via an overhead tank did not effectively overcome flow fluctuation problems, as the purported visual pattern distinctions were generally inconsistent and unconvincing. For example in Figure 3.7, the tracer patterns do not particularly show significant changes as the flow progresses from Trial 1 to 6 [from below to above equation (1) line in Figure 3.6]. The plot positions indicate that the flow structure of Trial 1, 2 and 3 should be of standard 2D closed rotational behaviour, while 3D helix-shaped patterns in the recirculation zone for Trial 4, 5 and 6. The movement of white ink roughly represented by red arrows upon entering the flows at the corner of the bifurcation are of similar angle and direction, i.e. straight downstream for Trial 1, 2 and 3 (standard 2D recirculation), while it seems to turning back upstream (3D helical) into the rotational zone for Trial 4, 5 and 6 – the latter set also display thicker ink cloud at upstream end. Nonetheless, the differences in flow patterns as the points approach the equation (1) separating line become less apparent, especially between Trial 3 and 4.





↓
main channel
flow direction

Figure 3.7 Video snapshots of flow conditions for trial runs 1 to 6 in the branch/lateral channel at 10 seconds after tracer injection



Therefore, the following can be concluded for the tracer injection method for visualizing flow structures:

- the distinction in ink patterns seems less obvious with the raising of weir heights (3 cm for Trial 1-6 and 9 cm for Trial 7-9), possibly due to deeper flows in the flume
- detailed scrutiny of the videos is required to conclusively detect differences in flow structures
- it is not effective nor consistent due to the following issues
 - flows in the channels are unsteady (although valves are fixed for steady flowrates) and turbulent – such conditions disrupt and smear the ink streams upon injection
 - somewhat clear differences for extreme points (lowest and highest flows) of all runs (A to E) – however, this diminishes with the increase in weir heights C_x , and C_y
 - nearer to the line, the ink patterns are too ambiguous for definite pattern recognition
 - considerable ink volume is released into the flow, this must be removed every so often via flushing to minimize obscuring water visibility in this closed system
 - the flow pattern identification process, done post experiments, is quite tedious and time-consuming

In addition to ink injection, manual (by hand) ‘introduction’ of paraffin flakes as well as small wood cuttings in and around the recirculation zone of the lateral/branch channel was also attempted, however these chips floated away downstream too easily, and were very much affected by local and periodic turbulence. For effective visualization (also would be the case later for PTV and LS-PIV runs), a considerably large amount was required thus rendering it cumbersome to execute as the material required ‘baskets’ at the lateral/branch and downstream tanks for collection and subsequent drying.

Accordingly, a new more reliable technique of flow visualization was then established based on assessment of GoPro videos recording the movement of a *long*

plastic strip or thread of visible colour, fixed to the same location as the previous ink injection point at downstream wall of the lateral/branch channel. As such, differences in the two identified flow structures of Momplot et al. (2017) , namely 2D standard closed recirculation versus 3D helical patterns, could be readily distinguished from apparent strip/thread behaviour. Preliminary results for Trial 7, 8 and 9 were promising based on video captures portrayed in Figure 3.8 and Figure 3.9. It can be discerned that 3D helical flow patterns as defined by Momplot et al. (2017) [based on computational rather than experimental methods], were reflected by twisty motion of the thread in Figure 3.9 as compared to it being relatively straight in Figure 3.8. Therefore, this procedure is adopted for visually identifying such flow structures in the experimental runs of Table 3.2, Table 3.3 and Appendix A, encompassing equal as well reduced channel widths over various weir heights.

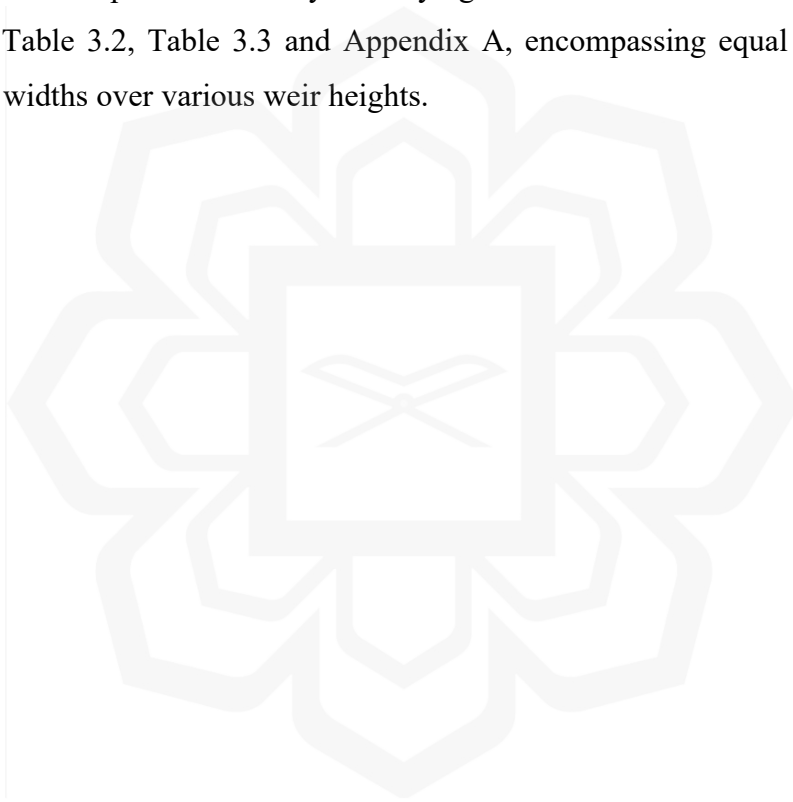




Figure 3.8 GoPro video snapshots of white thread movement (in four 1-second frames) for Trial 7 (Table 3.4) – 2D standard closed recirculation (Figure 3.6)

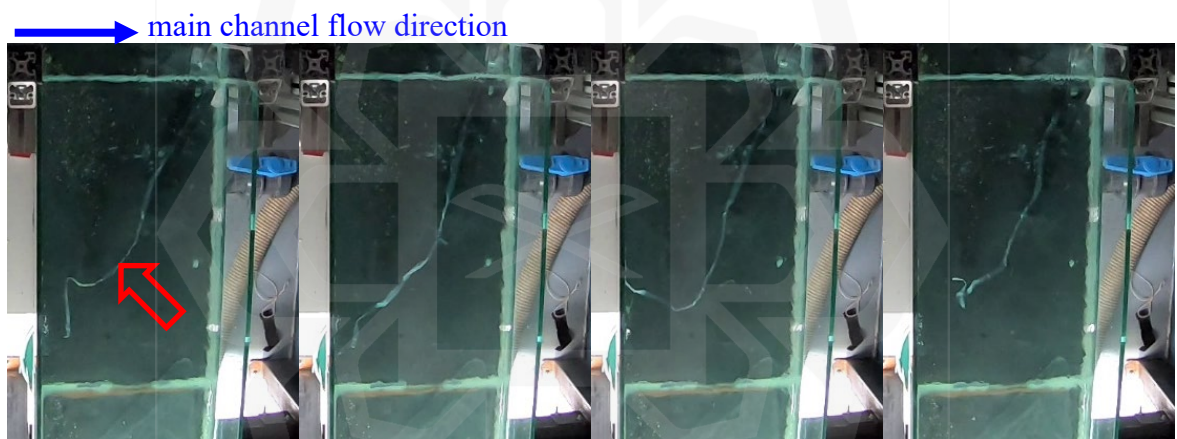


Figure 3.9 GoPro video snapshots of white thread movement (in four 1-second frames) for Trial 8 and 9 (Table 3.4) – 3D helical recirculation (Figure 3.6)

3.6 PARTICLE TRACKING AND IMAGE VELOCIMETRY

In addition to identification of flow patterns, the GoPro camera as installed in Figure 3.2 was also utilized for Particle Tracking Velocimetry (PTV) and Large-scale Particle Image Velocimetry (LS-PIV) analyses. These are described as follows:

- Particle Tracking Velocimetry (PTV)

This technique is implemented to precisely record surface patterns and velocities of the runs selected based on the pre-identified standard 2D closed or complex 3D helical flow structures in the recirculation zone. Early attempts of utilizing paraffin and wood chips indicate that these are suitable as tracer or seed material, however more ‘hot’ trials are required for confirmation. The lab and flume setup, with a Go-Pro video camera installed, is suitable with minimal fine-tuning necessary for this purpose.

- Large scale Particle Image Velocimetry (LS-PIV)

The same experimental configuration and tracers/seeds as in the PTV is to be utilized for LS-PIV. A possible difference is the material for seeding which could be confirmed with additional runs. The software chosen thus far for LS-PIV analysis once the video recordings are processed and verified is FUDAA, an open-source software co-developed and distributed by EDF and Irstea with DeltaCAD since 2010 (Le Coz et al., 2014). This approach has the advantage of not requiring a cumbersome and costly laser setup typical of standard PIV, thus suitable for in- and out-of-lab setups of measuring surface flows. As stated in the developer’s web-page, “The movement of tracers (natural or added) visible on the surface of the flow is detected by autocorrelation of patterns from one image to the next, after orthorectification of the images; the surface velocity fields thus obtained can be filtered and averaged over a series of image pairs to establish the average velocity field(.) (U)sing one or more bathymetric transects and coefficients linking surface velocity and depth-average velocity, one or more discharge estimates can (then) be calculated.” A screenshot of sample tracer movements recorded on video camera and analyzed on the FUDAA interface is presented in Figure 3.10.

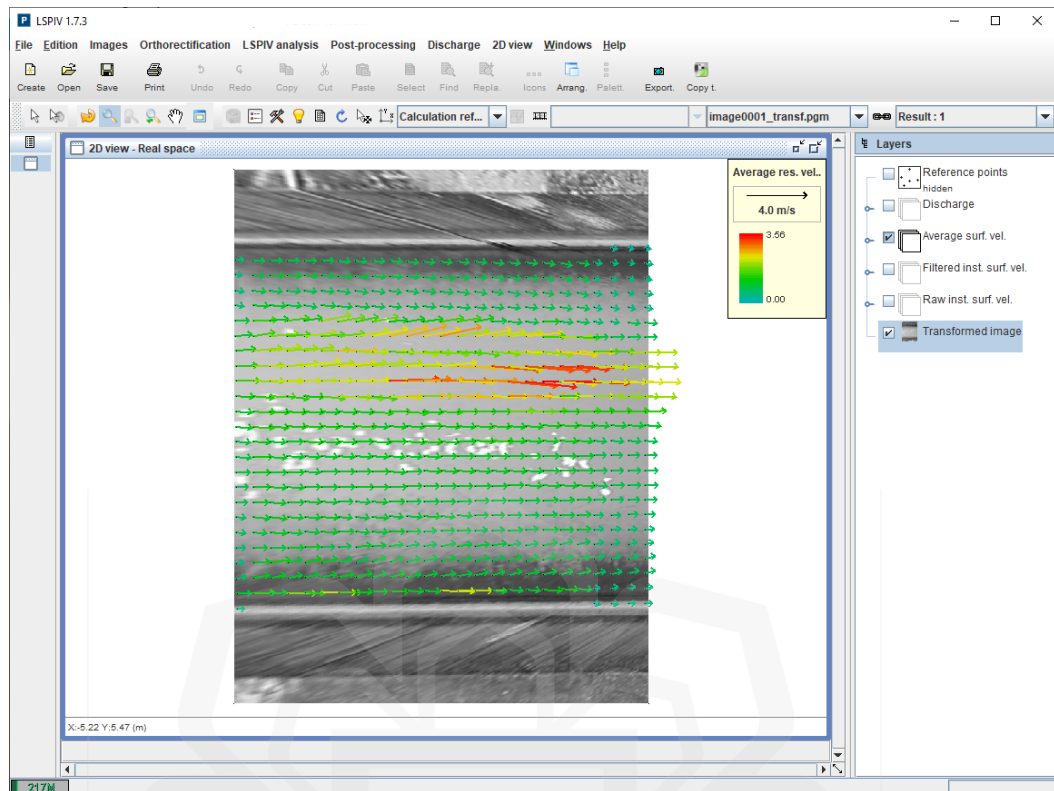


Figure 3.10 Screenshot of FUDAA interface showing resulting LS-PIV velocity vectors

Based on tests with paraffin and wood chips as tracers or seeds, it is decided that only certain runs utilizing the above methods would be executed, thus minimizing the problems associated with such material (especially wood chips) as highlighted earlier.

CHAPTER 4

RESULTS, ANALYSIS AND DISCUSSION

4.1 OVERVIEW

As described in Chapter 1, the division of open-channel flow at natural and man-made bifurcations poses many challenges in studying hydrodynamic and sediment transport processes at these geometrical singularities. The turbulent and complex structures resulting from flow separation complicate the quantification of water and sediment discharge rates in the downstream and branch channels. In this research, a series of experiments has been designed and conducted to extend the present body of work and datasets on flow in 90° (or T-shape) bifurcations of equal as well reduced branch width ratios – it is reiterated here that the work addresses all four of the stipulated objectives (more on this later).

Bifurcations in open-channel networks, naturally occurring or man-made, are geometrical singularities that result in separation of the inflows into typically two downstream branches. Many examples can be found in the riverine environment, stormwater and wastewater systems, as well as on flood streets and roads. In all cases, the main flow continues straight ahead, while the side branch, generally narrower, makes an angle close to 90° with the main channel, forming a junction also known as “T-shape bifurcation”. Moreover, bifurcations are naturally present in river networks, mostly upstream from islands and in deltas, where the main flow does not necessarily continue in a straight direction but may form a Y-shaped diverging fork that is not symmetrical a priori, at any angle, and with any branch width ratio.

In Chapter 2, information accessible from literature regarding both the flow processes and empirical discharge distribution laws at bifurcations is reviewed and examined in addressing the research objectives of Chapter 1. The experimental setup described in Chapter 3 is utilized to complete the available datasets compiled from previous investigations and test for recognition of flow patterns as well as discharge distribution correlations in both subcritical and transcritical flow regimes.

Therefore, the results for all experimental work in this investigation, namely (i) observation and identification of flow structures, and (ii) distribution of flow discharges from main to lateral/branch channels, are presented and analyzed in the following sections. Firstly, the experimental results and ensuing analysis for (i) attend to objectives 1 and 2 of this research, i.e.:

1. To determine experimental techniques of visualizing flow structures and distribution in bifurcated open-channel flow
2. To investigate methods to identify three-dimensional flow structures in bifurcated open-channel flow

Subsequently, (ii) then addresses the following objectives 3 and 4:

3. To improve empirical flow distribution relationships in open channel T-shape bifurcations
4. To establish the effect of reduced side branch width on empirical flow distribution relationships in open channel T-shape bifurcations

Note that the relevant variables and parameters required for this study have been identified and subjected to dimensional analysis in section 3.4.1, resulting with the derived dimensionless numbers as basis of the ensuing analysis.

4.2 OBSERVATION AND IDENTIFICATION OF FLOW STRUCTURES

As described in Chapter 3 (section 3.5), several techniques of visual identifying the 2D standard recirculation or 3D helical flow structures applied by Momplot et al. (2017) (Figure 2.9) have been tested for effectiveness and consistency. Based on video captures, both manual and tank-assisted ink injection methods were determined to be inadequate, while floating particles (paraffin and wood chips) were only useful for tracing patterns on the water surface. Upon execution of Trial 7, 8 and 9, it was then decided to adopt the usage of a *long plastic strip or thread of visible colour* to visually identify such flow structures in the experimental runs of Table 3.2, Table 3.3 and Appendix A. However, an outright confirmation of this procedure was not forthcoming based on a detailed assessment of the GoPro video recordings as the inferred strip behaviour was not consistently convincing although tendencies of each flow type were apparent. Thus, visual identification of structures and patterns in bifurcated open-channel flow in this investigation has been achieved but requires further reinforcement and affirmation. A supplementary approach which was explored in the first stage of experiments is deliberated as follows.

For the first stage, a total five sets of experimental runs, all in channels of equal 0.3-m width, have been conducted as listed in Table 3.2 with the results summarized in Table 4.1. Based on the practice of Momplot et al. (2017) as represented by Figure 2.9 in Chapter 2, $(F_{bx})^2$ values are plotted versus b_x/h_u in Figure 4.1. As can be discerned, there is a noticeable difference in slopes or curvature of the data points depending on their location above or below equation (2.1), reintroduced as the following equation (4.1), developed by the previous researchers [equation 6 of Momplot et al. (2017)]:

$$(F_{bx})^2 = 0.047 \frac{b_x}{h_u} - 0.094 \dots\dots\dots (4.1)$$

[equation 6 of Momplot et al. (2017)]

This is apparently caused by the two distinctive flow structures determined from positions of plotted run results relative to the linear relationship of equation (1). In the earlier work, it was observed and ascertained that standard 2D closed rotational

behaviour occurred in the recirculation zone of the lateral/branch channel for conditions that appear below the line in Figure 4.1. Conversely, 3D helix-shaped patterns would be evident for the region above this 'equation'. The steeper curves of data points, regardless of equal downstream and lateral/branch weir heights, seem to indicate more complex structures of the latter, occurring at higher discharges (deeper depths) and corresponding Froude numbers.

A further check on such trends is carried out by estimating the change in fluid or hydraulic power (defined as the rate of energy transfer) when the flow of each run is split between the main and branch or lateral channels. The full energy equation of flowing water in conduits comprises of elevation, pressure and velocity heads plus any external source (pump), then minus major and/or minor losses (friction, expansion/contraction) and any external sink. In studying bifurcated smooth open-channel flow for this research, it is expected that rate of energy is transferred via changes only in elevation, pressure (both represented by water depth for free-surface flows) and kinetic (velocity) terms caused by such division. This is reflected in the power (each total head multiplied by flow depth) lost as flow travels from the main to downstream and lateral/branch channels.

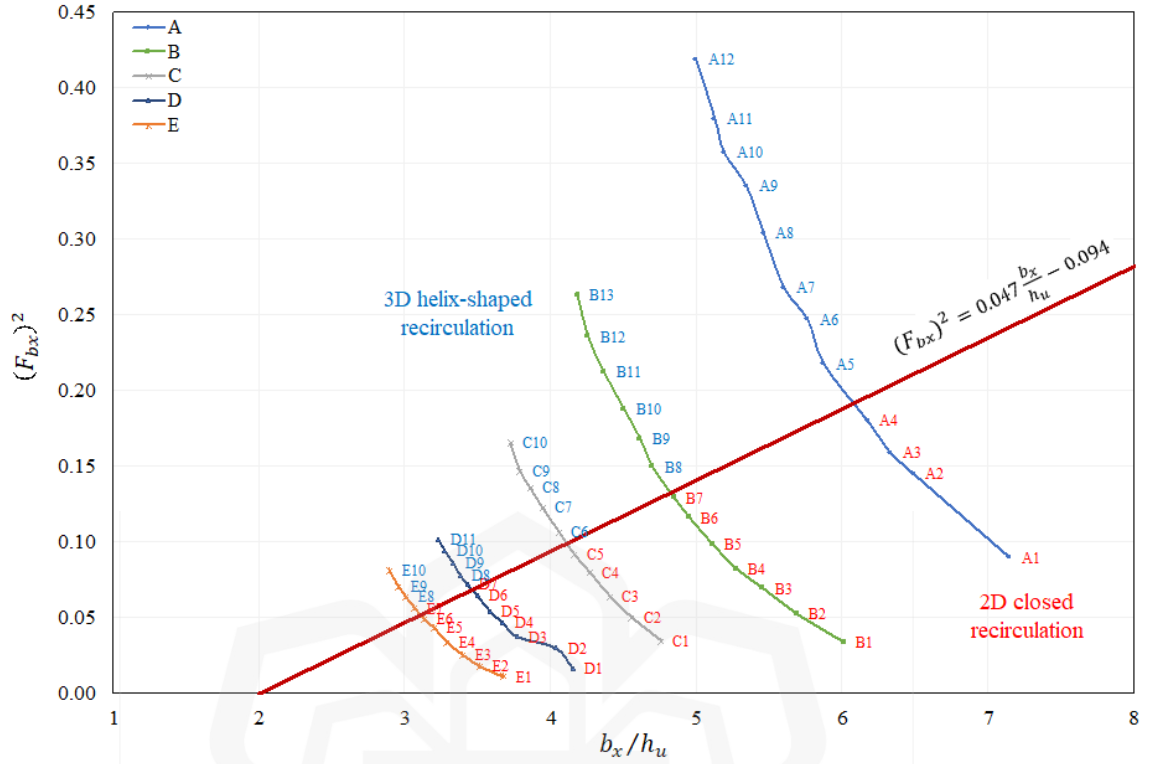


Figure 4.1 $(F_{bx})^2$ versus b_x/h_u for runs A, B, C, D and E

The trend of power loss (in percentage) after the bifurcation can then be plotted with respect to upstream (total) discharge Q_{in} , as in Figure 4.2. Due to the small initial discharges between 2 to 6 l/s in the testing flume and limited by the equal weir heights (3 to 7 cm) at the two channel outlets, the computations of fluid power as in the main and branch channels were found to be highly sensitive to very small variations in measured water depths. Compounded by the possible deviation of the kinetic energy correction factor beyond 1, the expected loss in power (generally <1%) through the bifurcation was not evident at low Froude numbers. This discussion then focuses on the observed trend rather than actual power losses.

For run set A (Table 3.2 and Table 4.1), with weir heights fixed at 3 cm for both outlets, it is observed that power loss (%) after the flow bifurcates is lowest at the starting discharge of 2.42 l/s (run A1). As flow increases, this increases until the highest loss at 8.94 l/s (run A12). Based on the above discussion for Figure 4.1 and the derived relationship [equation (6)] of Momplot et al. (2017), it is apparent that at a particular % level (or limit) of power loss between run A4 and A5, the flow structure changes from

standard 2D closed to 3D helical patterns. This limiting feature seems to remain the same (drawn as a straight horizontal line in Figure 4.2) for all the other sets (B, D and E) except for C. Overall, 56 out of a total of 66 plotted positions in Figure 4.2 exhibit this characteristic in accordance to the flow structures (i.e. above or below the limit line) indicated by Figure 4.1.

The observed trend in fluid power is most likely caused by the higher rate of energy loss as the flow bifurcates to the downstream areas with the increase in discharge (and Froude no.). At a specific %-loss limit, the standard 2D closed patterns initially occurring in the recirculation zone of the lateral/branch channel are ‘converted’ to more complex 3D helixes, the latter apparently generated by more power. As flow becomes progressively more turbulent, the patterns in the lateral/branch channel are transformed from standard somewhat tranquil 2D shapes at low discharges and corresponding Froude numbers to more intricate 3D structures at higher values. This indicates the formation of vortices and helixes that require more energy, similar to the energy-cascade concept in turbulent flow, thus resulting in the loss of fluid power beyond a certain transformation point. Since it is consistently at the same level for most of the runs (except set C due to possibly measurement errors), this %-loss limit can be considered *the* threshold to distinguish between the two distinct structures in such bifurcated open-channel flow conditions. At this juncture of the work, more analysis is required to overcome the sensitivity of the measurements at low discharges that affect the precise quantification of power loss and the aforesaid threshold.

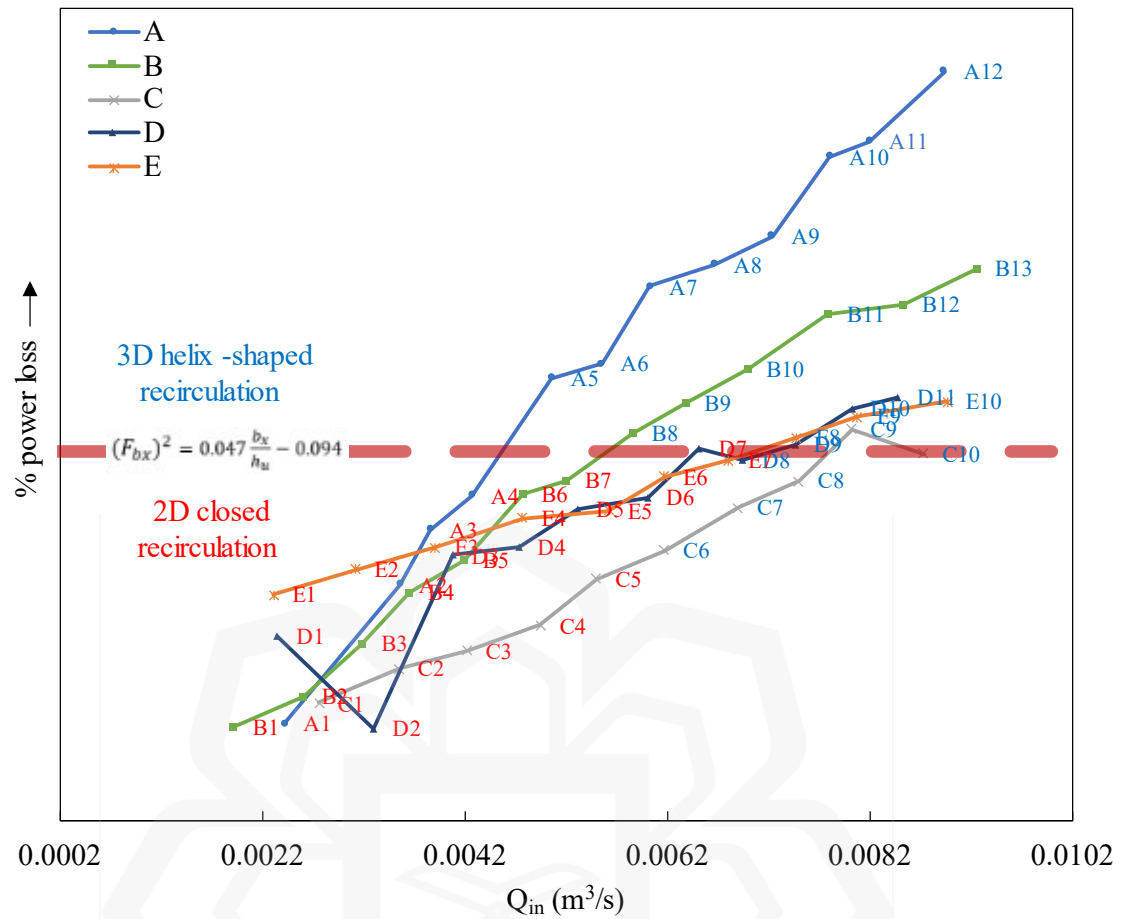


Figure 4.2 Trend of fluid power loss (versus Q_{in}) as flow divides downstream and into the bifurcation for runs A, B, C, D and E

4.3 DISTRIBUTION OF FLOW DISCHARGES FROM MAIN TO LATERAL/BRANCH CHANNELS

4.3.1 Experimental results

A total of six hundred and sixty-eight (668) configurations were measured in either subcritical regime everywhere (including in the side branch) by sufficiently raising the side branch weir crest (C_y) or transcritical regimes when this weir crest was lowered (Table 4.1). As described in Chapter 3, these runs comprised of equal widths ($b_x, b_y = 30$ cm) as well as reduced side branch widths (b_y) of 10, 18, 22 and 26 cm. Moreover, the horizontal channel alignment and minimum elevation of the downstream weir crest (C_x) ensured that no flow fell into supercritical regime in the upstream branch (Rivière and Perkins, 2004), which is beyond the scope of present work. This is confirmed by the ranges of F_{xo} attained in the experiments as listed in Table 4.2. with the results presented in Table 4.3 and Appendix B.

Table 4.1 Experimental configurations of bifurcated flow

R_b	Re_{bx}	F_{xo}	F_{yo}	number of measurements
1	$[5.6 - 30.8] \times 10^3$	0 – 0.38	0.02 – 0.73	132
26/30	$[7.9 - 30.0] \times 10^3$	0 – 0.37	0.01 – 0.71	126
22/30	$[7.8 - 32.1] \times 10^3$	0 – 0.40	0.01 – 0.68	132
18/30	$[8.6 - 30.7] \times 10^3$	0 – 0.35	0.02 – 0.68	142
10/30	$[8.8 - 31.4] \times 10^3$	0 – 0.36	0.03 – 0.65	136
Total				668

The experimental data of Table 4.3 and Appendix B are further processed for assessing the discharge distribution formulations of bifurcated flows established by Rivière et al (2007) (third objective of this research).

As reiterated and recapped from the literature review of Chapter 2, Rivière et al. (2007) investigated the general problem of free-surface flows divided through an intersection of four equal-width open channels in a laboratory setting, which is the foundation for Momplot et al. (2017) and this study. For sub-critical (Froude number not exceeding 0.8) conditions in the 3-branch bifurcation configuration, the following equations (with notations adjusted for consistency) were established:

$$h_d = C_x + h_{crit,xo} \left[1 + 0.793 \left(\frac{C_x}{b_x} \right) \right] \dots\dots\dots (4.2a)$$

similarly, for lateral or branch channel

$$h_l = C_y + h_{crit,yo} \left[1 + 0.793 \left(\frac{C_y}{b_y} \right) \right] \dots\dots\dots (4.2b)$$

$$F_{Cx} = \frac{h_{crit,x}}{C_x^{3/2}} \dots\dots\dots (4.3)$$

$$R_{q-sym} = 0.5 - 0.04675 F_{Cx} \dots\dots\dots (4.4)$$

$$R_{q-sub} = R_{q-sym} + 0.358(1 - R_c) F_{Cx}^{-0.77} \dots\dots\dots (4.5)$$

Equations (4.2a) and (4.2b) are the stage-discharge relationships or rating curves for the downstream and lateral/branch channels, respectively, derived and calibrated empirically based on weir heights (Rivière et al., 2007).

For the flows tested as in Table 4.3 and Appendix B, the corresponding depths are computed and plotted against discharges in Figures 4.3 to 4.12. It is evident that these fit very well with experimental measurements of downstream as well as lateral/branch depths for a majority of the runs. Therefore, the development of equations (4.2a) and (4.2b) in estimating flow discharges based on depths by Rivière et al. (2007) is backed by the completed datasets. Furthermore, upon grouping them together in Figures 4.13 and 4.14, the downstream and lateral/branch stage-discharge relationships (rating curves) based on measured depths reveal identical trends, offset by their respective weir heights, C_x or C_y .

The distribution of flow in the lateral or branch channel, with respect to upstream or main, is estimated with equation (4.5), itself dependent on equations (4.3) and (4.4). Upon completing the required calculations for all the experimental runs, R_{q-sub} is

graphed versus F_{Cx} in Figure 4.15. Note that since the flows are over equal-height weirs ($C_x = C_y$) at outlets of both downstream and lateral/branch channels (a condition resolved in Rivière et al. (2007) and Momplot et al. (2017) to reduce the number of parameters for dimensional analysis), R_{q-sub} then equals R_{q-sym} . In Figure 4.7, the fitted relationship in blue is derived from equation (4.5) computed using measured values, while all the actual data in red are located below it. Trend wise, both sets agree quite well with the expected decrease in discharge distribution into the lateral/branch channel as F_{Cx} (Froude number based on main channel downstream weir height) intensifies with rising inflow.



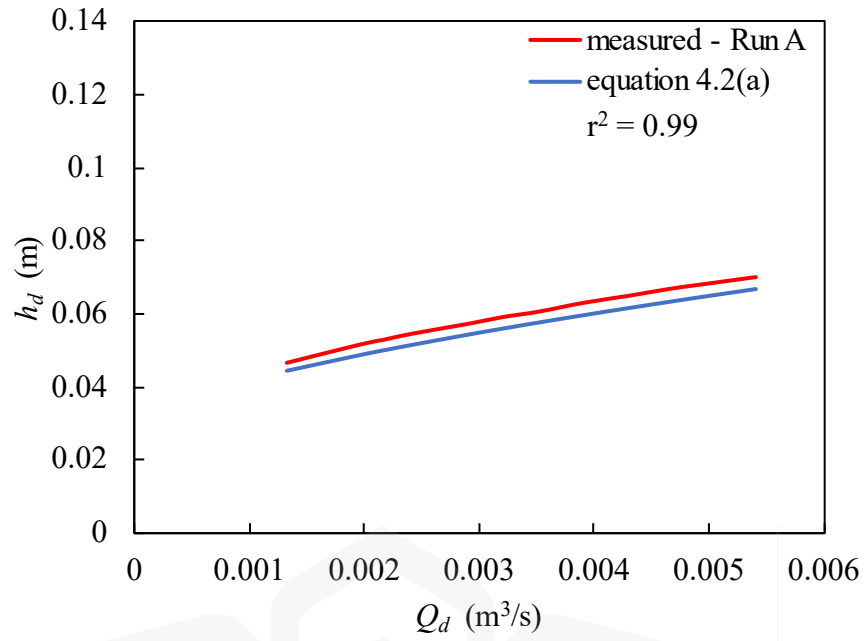


Figure 4.3(a) Downstream stage-discharge plots for measured and computed flow depths [equation 4.2(a)] – run A in Table 3.2 ($C_x = C_y = 30$ mm)

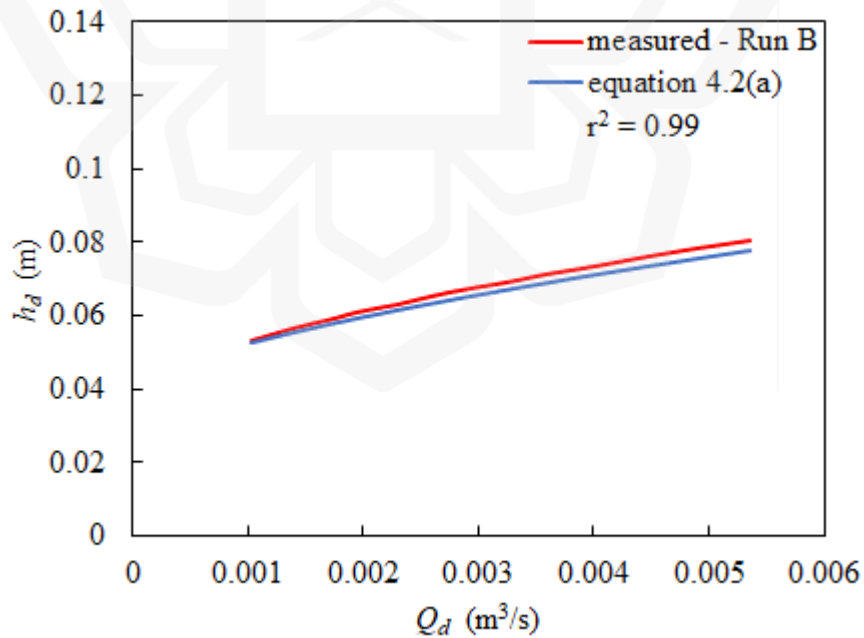


Figure 4.3(b) Downstream stage-discharge plots for measured and computed flow depths [equation 4.2(a)] – run B in Table 3.2 ($C_x = C_y = 40$ mm)

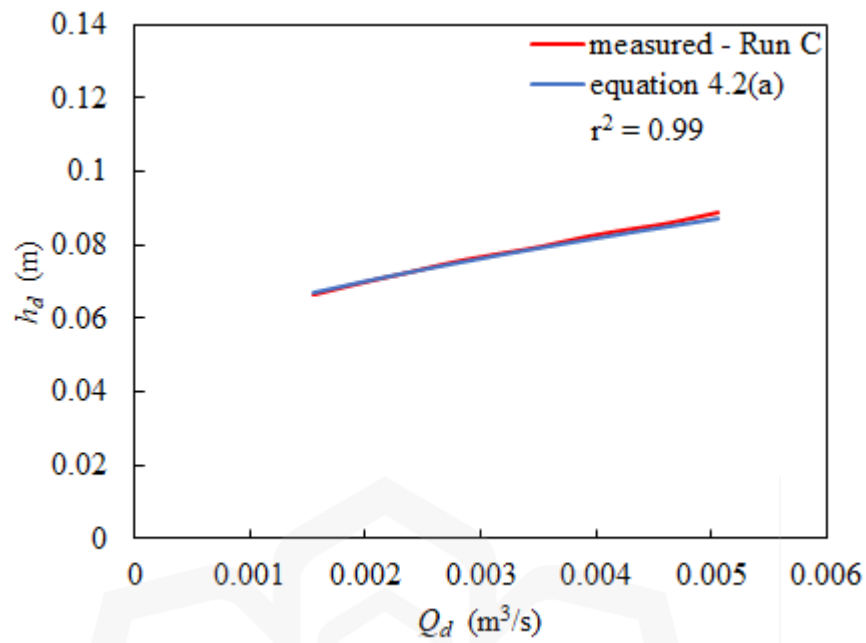


Figure 4.3(c) Downstream stage-discharge plots for measured and computed flow depths [equation 4.2(a)] – run C in Table 3.2 ($C_x = C_y = 50$ mm)

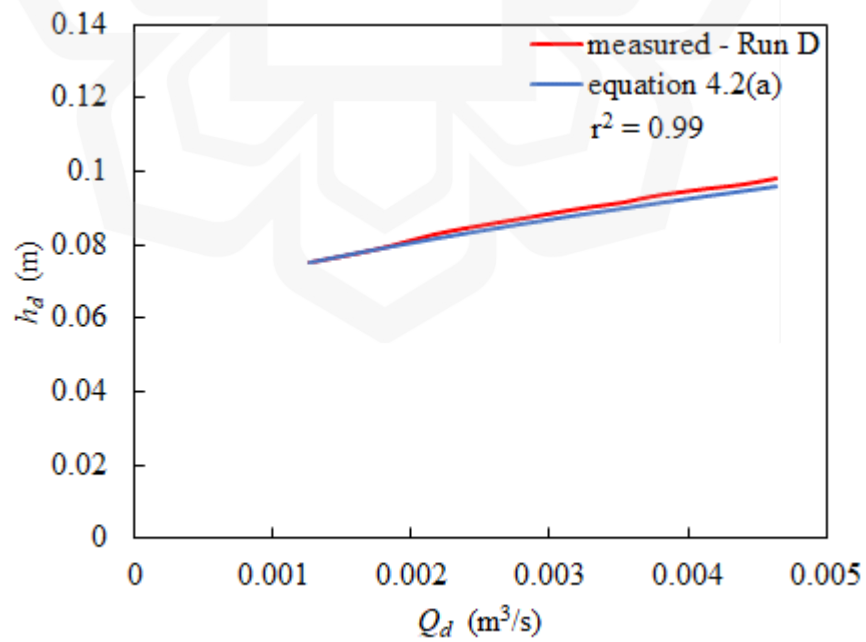


Figure 4.3(d) Downstream stage-discharge plots for measured and computed flow depths [equation 4.2(a)] – run D in Table 3.2 ($C_x = C_y = 60$ mm)

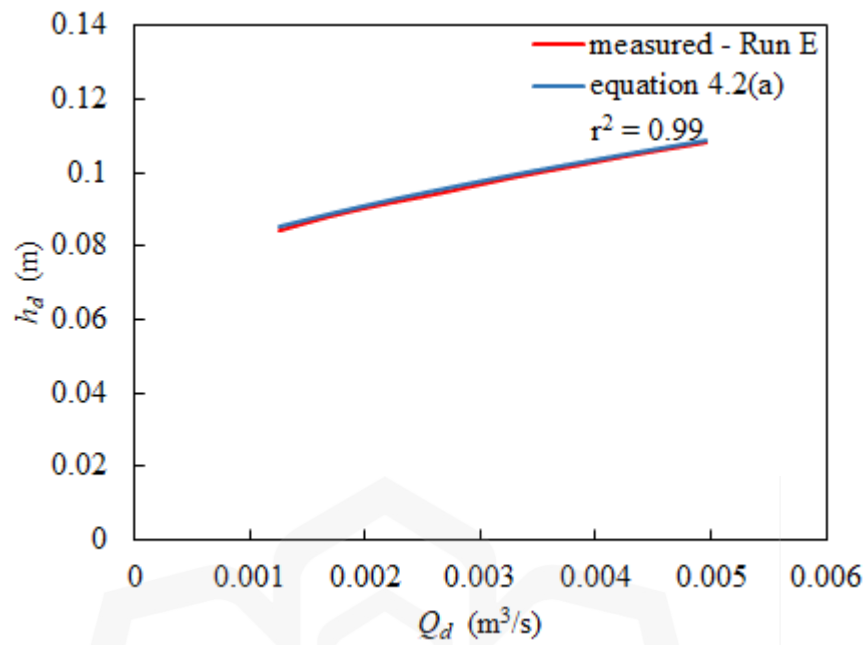


Figure 4.3(e) Downstream stage-discharge plots for measured and computed flow depths [equation 4.2(a)] – run E in Table 3.2 ($C_x = C_y = 70$ mm)

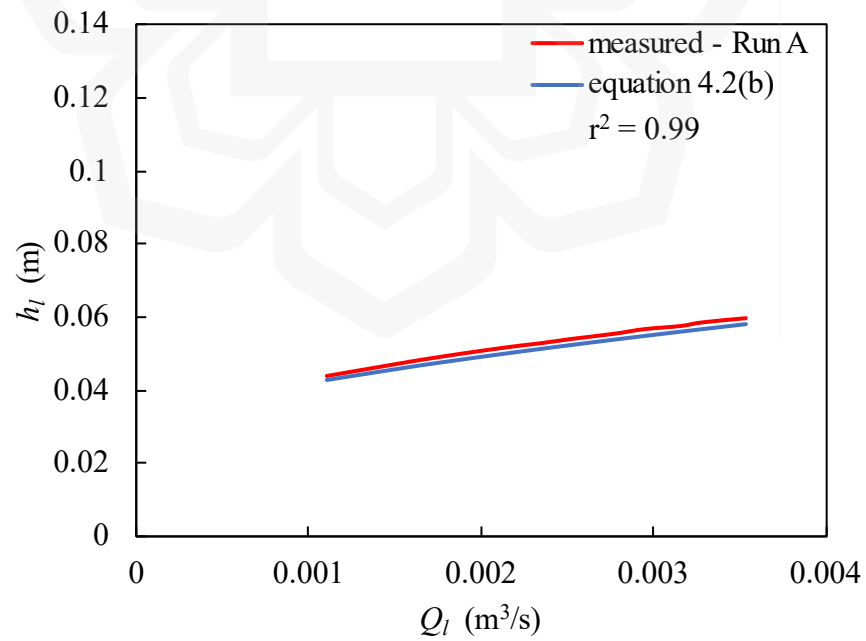


Figure 4.4(a) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.2(b)] – run A in Table 3.2 ($C_x = C_y = 30$ mm)

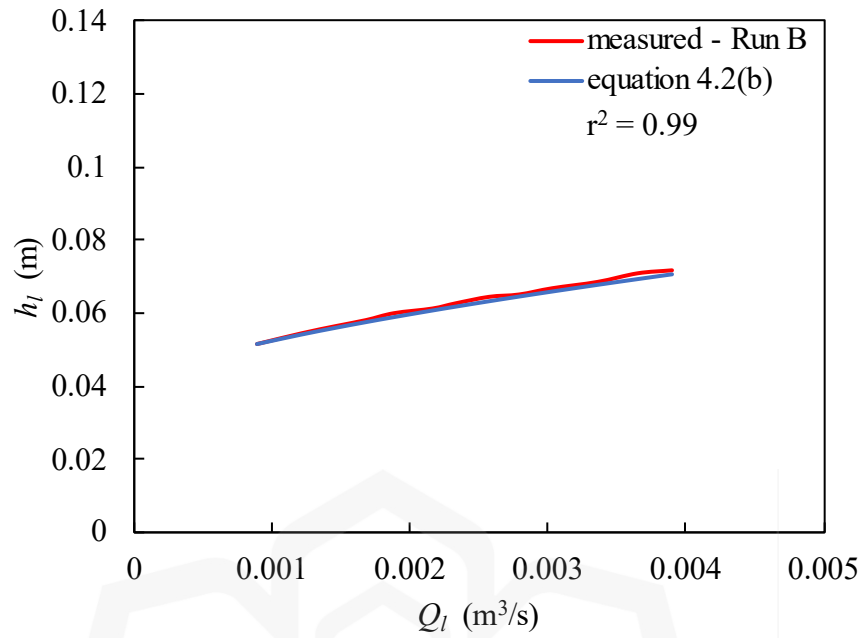


Figure 4.4(b) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.2(b)] – run B in Table 3.2 ($C_x = C_y = 40$ mm)

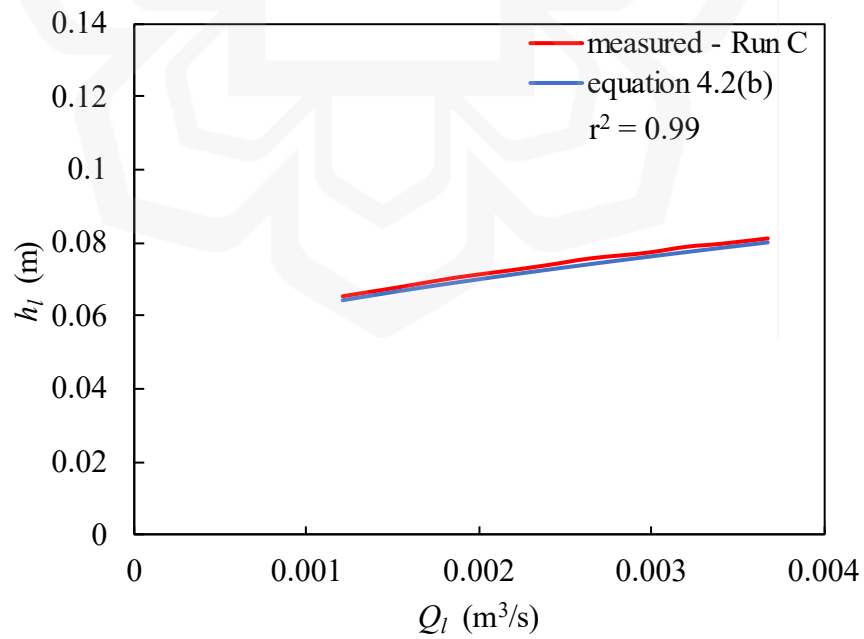


Figure 4.4(c) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.2(b)] – run C in Table 3.2 ($C_x = C_y = 50$ mm)

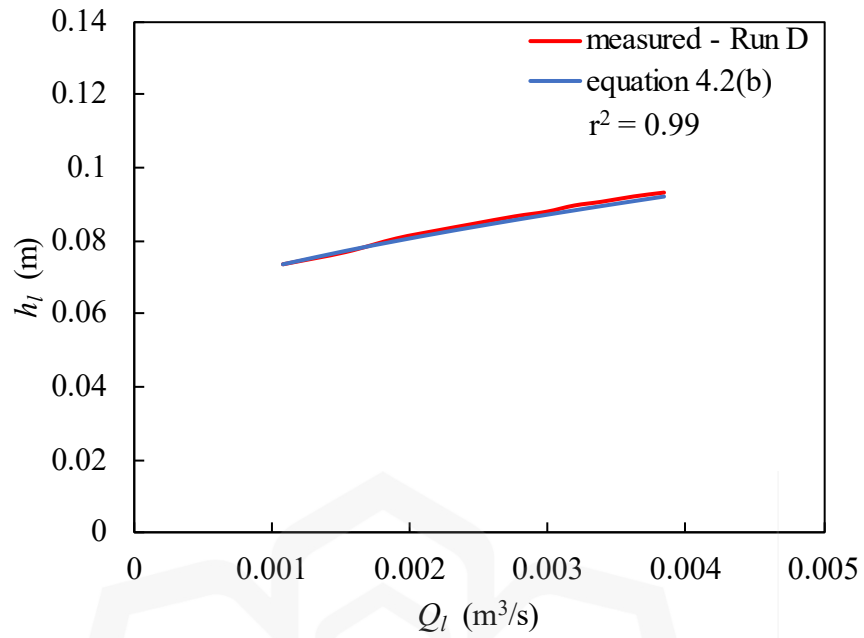


Figure 4.4(d) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.2(b)] – run D in Table 3.2 ($C_x = C_y = 60$ mm)

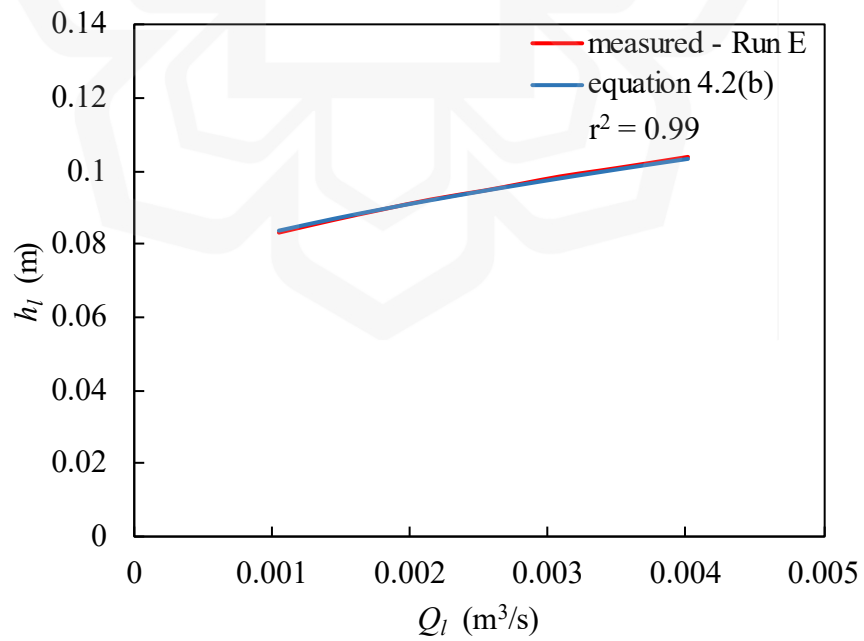


Figure 4.4(e) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.2(a)] – run E in Table 3.2 ($C_x = C_y = 70$ mm)

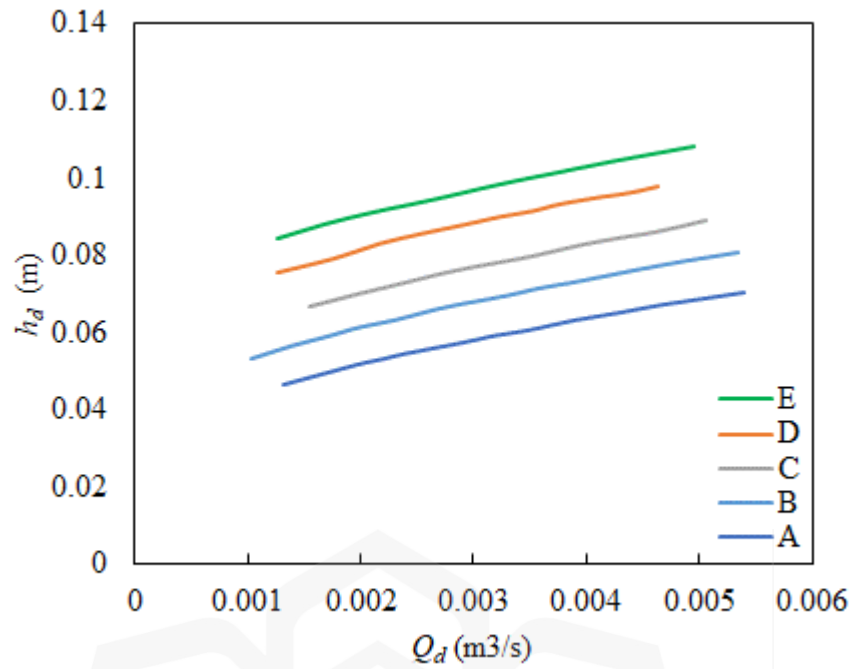


Figure 4.5 Downstream stage-discharge plots for measured flow depths (Runs B to E)

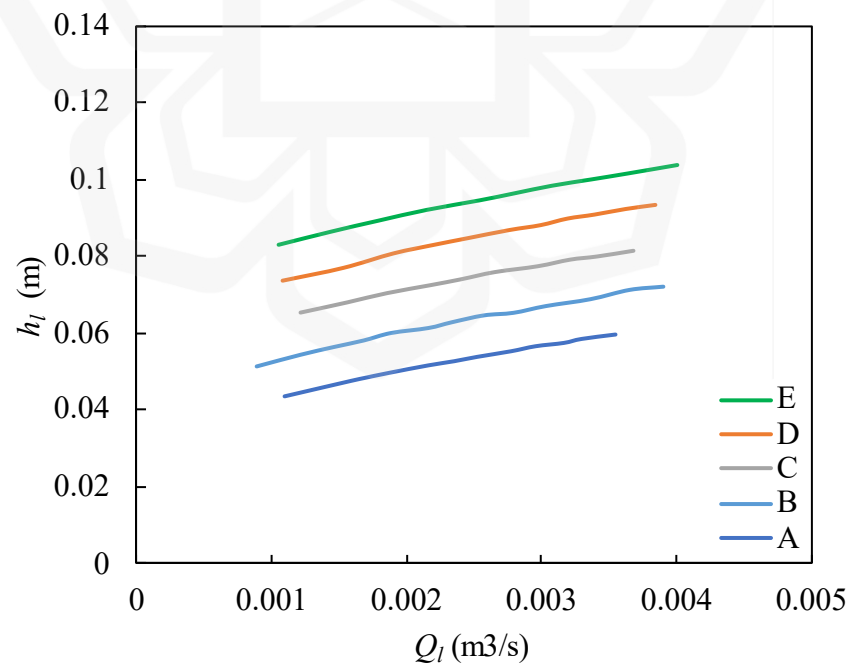


Figure 4.6 Lateral/branch stage-discharge plots for measured flow depths (Runs B to E)

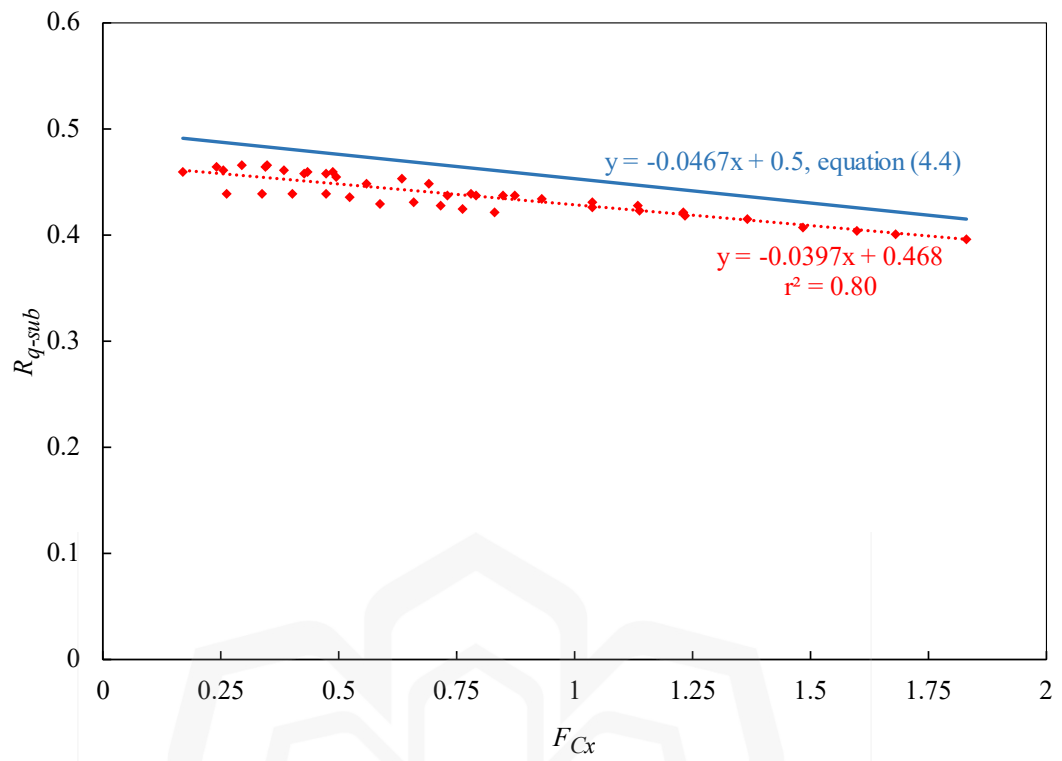


Figure 4.7 Flow distribution (R_{q-sub}) versus F_{Cx} : [equation (4.5)] and experimental data of runs A to E

4.3.2 Discharge distribution – subcritical regime

In subcritical regime throughout both channels, the two discharge distribution correlations available in the literature, proposed by Hager (1991), Hager (2010) and Rivière et al. (2007), are evaluated and discussed based on present as well as past measurements. According to the state of present knowledge (§2.5), the lowest threshold value of Froude number in the side-branch (F_{yo}) reported in the literature to separate the subcritical and transcritical regimes is around 0.3. Thus, to ensure that all flows are in subcritical regime, only experiments with $F_{yo} < 0.3$ are retained for this analysis.

Firstly, based on equation (4.5), Hager (1991) adjusted the measurements performed by Lakshmana Rao et al. (1968) as follows:

$$R_q = 1 - \tanh[5(A - F_{yo})F_{xo}^{1/2}] \quad (4.6)$$

where $A = 0.56$ for $R_b = 0.5$ being the only branch width ratio tested by Lakshmana Rao et al. (1968). Note that equation (4.6) does not consider any impact of F_{bx} on R_q . To extend the validity of equation (4.6) to R_b values other than 0.5, the best fitted values of coefficient A are obtained for the five R_b runs of present experimental campaign and plotted in Figure 4.16 (insert), along with the value $A = 0.56$ for $R_b = 0.5$ proposed by Hager (1991) and Hager (2010). A monotonic decreasing trend is observed, but the value proposed in Hager (1991) slightly departs from the adjusted curve $A = 0.468R_b^{-0.306}$. It is suggested here that this slight difference is due to the measurement position of F_{xo} at the entrance of the downstream branch as performed by Lakshmana Rao et al. (1968), while it is located further downstream in the present experiments as indicated in Figure 3.2.

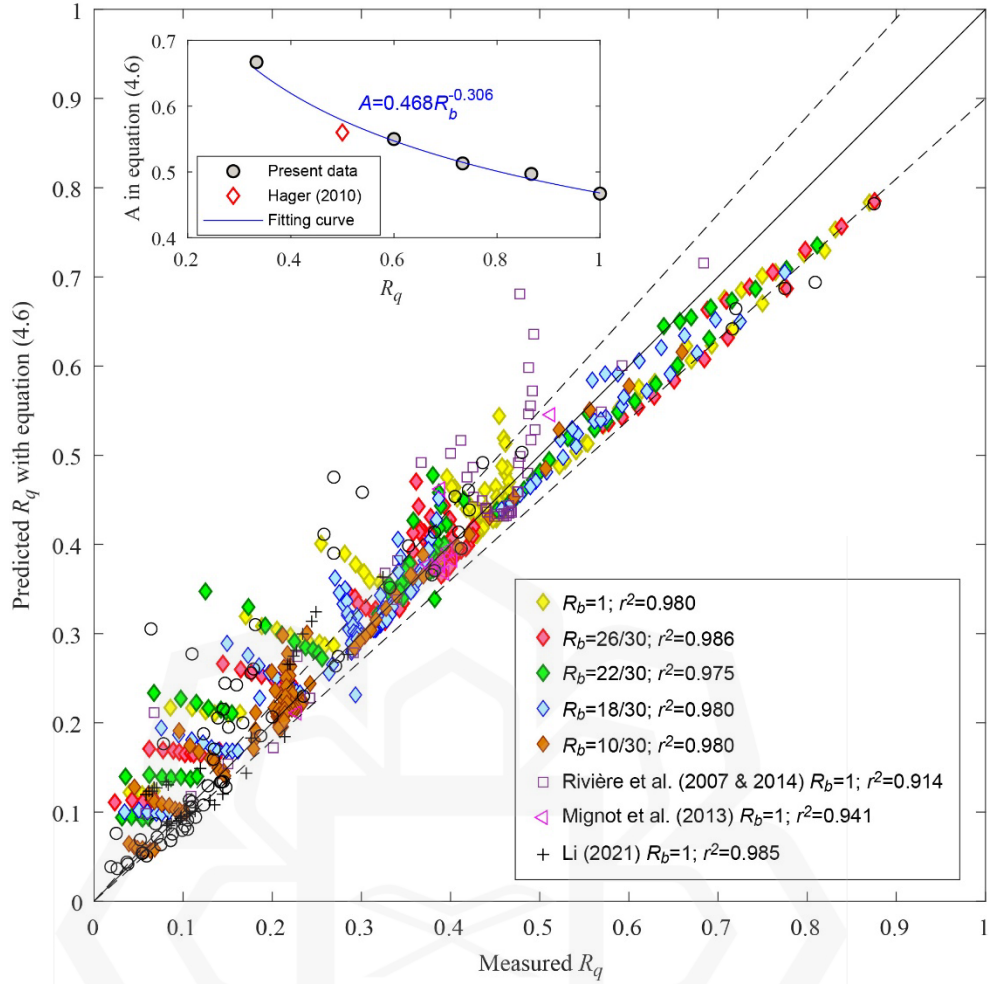


Figure 4.8 Predicted discharge distribution in subcritical regime with equation (4.6) for all available flow configurations with $F_{y0} < 0.3$; line for exact match ($y = x$) is solid while +/- 10% are dashed. Insert: best-fitted A curve for the 5 tested R_b values of present experiments (blue) and $A = 0.56$ for $R_b = 0.5$ (red) as proposed by Hager (1991) when adjusting data from Lakshmana Rao et al. (1968).

Finally, the discharge distributions R_q predicted by equation (4.6) combined with the adjusted curve for A values are compared in Figure 4.8 with the measurements for all available flow configurations in subcritical regime (ensured here by $F_{y0} < 0.3$). A relatively fair agreement is obtained for all R_b values with r^2 coefficients of determination exceeding 0.9. Yet, Figure 4.8 indicates that the agreement is not perfect as many points fall out of the +/- 10% range (most exceeding +100% for $R_q < 0.15$)

with a tendency of equation (4.6) to overestimate the lowest R_q values and underestimate the largest. Note that, in this figure and the rest that follow, two doubtful test data labelled #5 and #12 by Law (1965) [$R_q = 0$ or 1] were excluded due to their ‘outlier’ plotting positions.

More recently, utilizing weir crest heights C_x and C_y , Rivière et al. (2007) used equation (3.4) based on the dimensional analysis stipulated in §3.4.1 to propose the following empirical relationship for subcritical flow configurations (restricted to $R_b = 1$ and neglecting the effect of C_x/b_x):

$$R_q = 0.5 - 0.04675F_{Cx} + 0.358 \left(1 - \frac{C_y}{C_x}\right) F_{Cx}^{-0.77} \quad (4.7)$$

Note that equation 4.4 is the reduced from equation (4.7) for the case when both outlet weirs are of the same height, i.e. $C_x = C_y$. The agreement with the authors’ experimental campaign exceeded that of equation (4.6) with $r^2 = 0.99$. However, as divulged earlier, this correlation is not adapted to field conditions where downstream weirs are generally not available for use, and does not include the effect of the branch width ratio. Another empirical correlation is then sought, involving the dimensionless parameters of equation (3.5) instead of that of equation (3.4). As it is extremely difficult to adjust independently these parameters in experiments, (4.7) is used to generate synthetic data for various values of the 3 Froude numbers (F_{bx} , F_{xo} , F_{yo}). Several different series are generated, each with one of the three parameters varying while the other two are kept constant. Utilizing MatLab, this allows the identification of best functions reproducing the influence of each parameter of equation (3.5). Consistent with equation (4.6) of Hager (1991), F_{bx} shows a very limited influence on the discharge distribution (not shown here) and is discarded. Empirical coefficients are then finely adjusted with present experimental results, using the least squares method, including the effect of the branch width ratio. The final empirical correlation, equation (4.8), provides the T-shape bifurcated discharge distribution in subcritical flow regime for R_b ranging between 1/3 and 1:

$$R_q = a \ln \left[\frac{b[\exp(1/a)-1]\left\{\frac{R_b F_{yo}}{F_{xo}}\right\}}{[\exp(1/a)-1]+b\left\{\frac{R_b F_{yo}}{F_{xo}}\right\}} + 1 \right], \quad a = 1.012 \text{ \& } b = 0.941 \quad (4.8)$$

where R_q tends towards 0 (or 1) as F_{yo} (or F_{xo}) approaches 0. The major influence of the dimensionless parameter $\frac{R_b F_{yo}}{F_{xo}}$ is confirmed by Figure 4.9(a) with a very narrow dispersion of measured discharge distributions when plotted against it. Moreover, the validity of equation (4.8) is depicted on Figure 4.9(b), revealing a better agreement with measurements than equation (4.6) of Figure 4.8, with coefficients of determination r^2 reaching 0.99 for all considered datasets and R_b values.

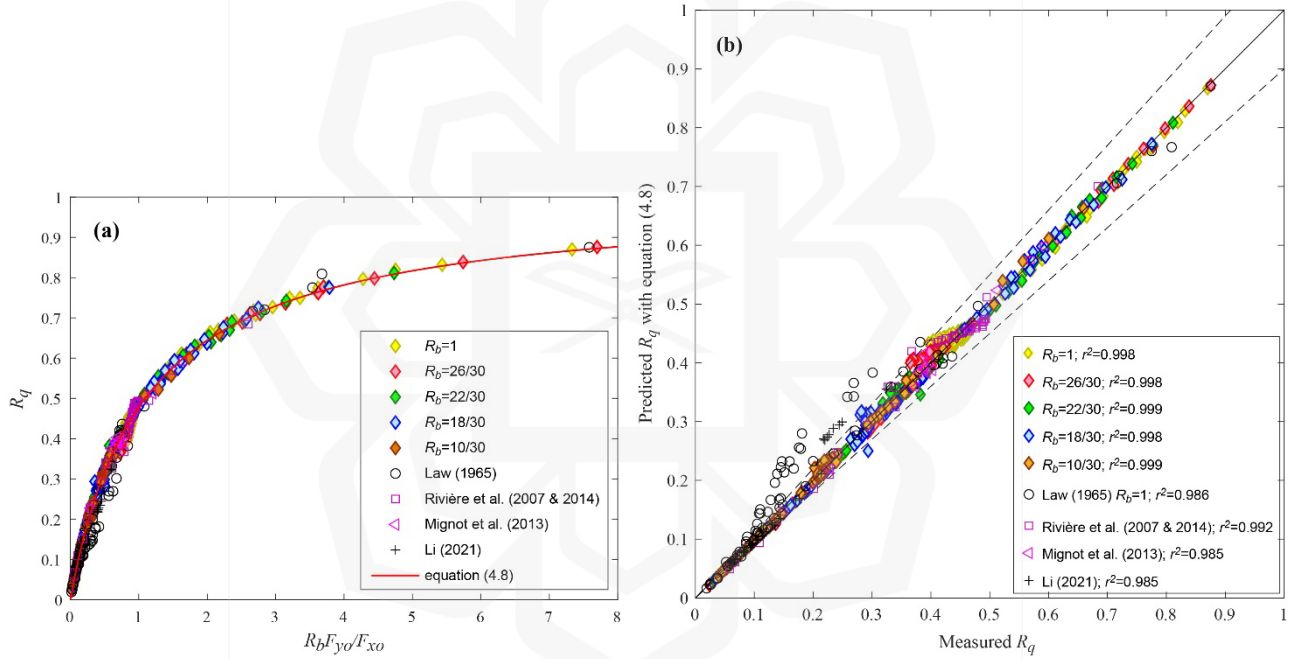


Figure 4.9 Comparison of measured vs equation (4.8) predicted discharge distribution in subcritical regime for all available flow configurations with $F_{yo} < 0.3$, (a) as a function of the dimensionless number $\frac{R_b F_{yo}}{F_{xo}}$, and (b) compared with exact match ($y = x$) as solid and $\pm 10\%$ as dashed lines.

4.3.3 Discharge distribution – free recirculation transcritical regime

The discharge distribution correlations adapted to the “free recirculation” pattern of the transcritical regime expounded by Rivière et al. (2014), also termed “free flow in the branch channel” by Lakshmana Rao et al. (1968), are evaluated here. In this regime, the downstream boundary of the side branch (due to supercritical conditions after the *vena contracta*) has no impact on the discharge distribution in the bifurcation. To ensure that all flows from the present experiments belong to this regime, the following procedure is followed. For fixed inflow Q_{xi} and downstream weir crest height $C_x = 5$ cm, the height of the side branch weir crest C_y is regularly lowered (thus increasing R_q and the Froude number in the side branch F_{yo}) until the discharge distribution remains stable, as depicted in Figure 4.10. Therefore, all flow configurations on this plateau of constant R_q (on the right side of the dashed lines representing the threshold F_{yo} in Figure 4.10) are assumed to belong to the free-recirculation transcritical regime and used henceforth in the analysis. It can be discerned that the threshold F_{yo} varies between about 0.35 to 0.65 for $R_b = 1$ but the approximate range narrows closer to 0.6 with decreasing width ratio. This tallies with as well as supplements the findings of previous researchers as reviewed earlier from the literature (§2.5).

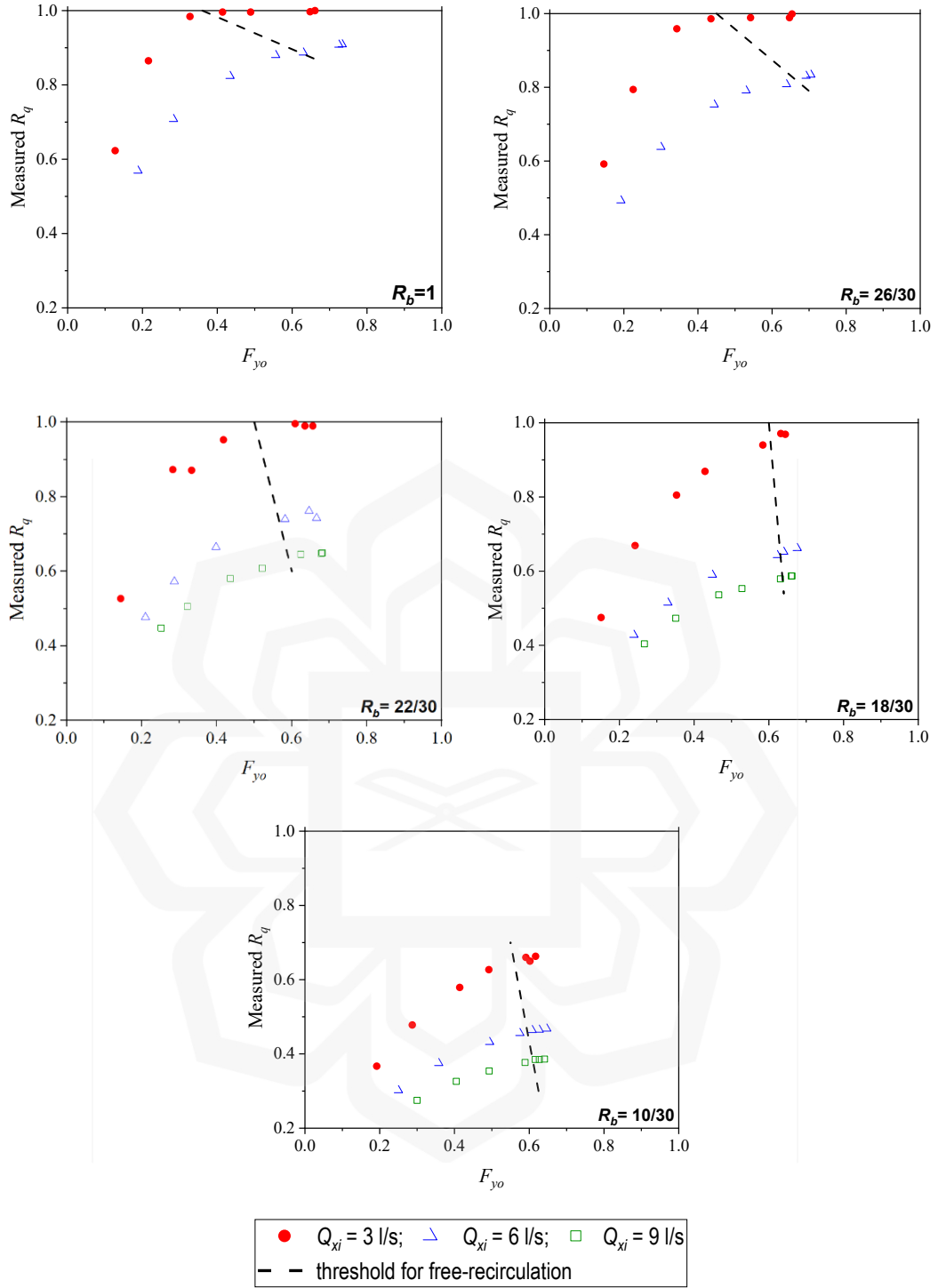


Figure 4.10 Analysis of the subcritical-transcritical regimes transition for the 5 tested R_b configurations. In each series, Q_{xi} and $C_x = 5 \text{ cm}$ are maintained constant while lowering C_y , thus increasing F_{yo} . The right most points for all series correspond to $C_y = 0$. The dashed lines demarcate the assumed limits between the free- and drowned-recirculation transcritical regimes.

Based on an extended experimental campaign, Lakshmana Rao & Sridharan, (1967) proposed a discharge distribution correlation in this free-recirculation transcritical regime (with $C_y = 0$), based on equation (3.5) by neglecting the influence of F_{bx} and F_{yo} as follows:

$$\log_{10}(R_q) + KF_{xo}^\alpha = 0 \quad (4.9)$$

where K and α are constant coefficients that are dependent on the bifurcation angle and width ratio, respectively. For a T-shaped bifurcation (90° angle) and $R_b = 1$, Lakshmana Rao & Sridharan, (1967) proposed $K = 1.11$ and $\alpha = 0.995$. Based on their measurements, Rivière et al. (2014) recommended adjusting the coefficients to $K = 0.925$ and $\alpha = 0.98$ (instigated by the discrepancy between the purple or violet square symbols and $y = x$ line in Figure 4.11). However, the best agreement with the present data was obtained with Lakshmana Rao & Sridharan, (1967) coefficients for $R_b = 1$, which were thus retained in this analysis. Moreover, Lakshmana Rao & Sridharan, (1967) proposed fitting coefficients for three additional R_b values: $K = 1.4$ and $\alpha = 0.672$ for $R_b = 0.25$; $K = 1.27$ and $\alpha = 0.86$ for $R_b = 0.5$ and $K = 1.16$ and $\alpha = 0.938$ for $R_b = 0.75$, which are used here to establish a discharge distribution law valid for a wide range of branch width ratios.

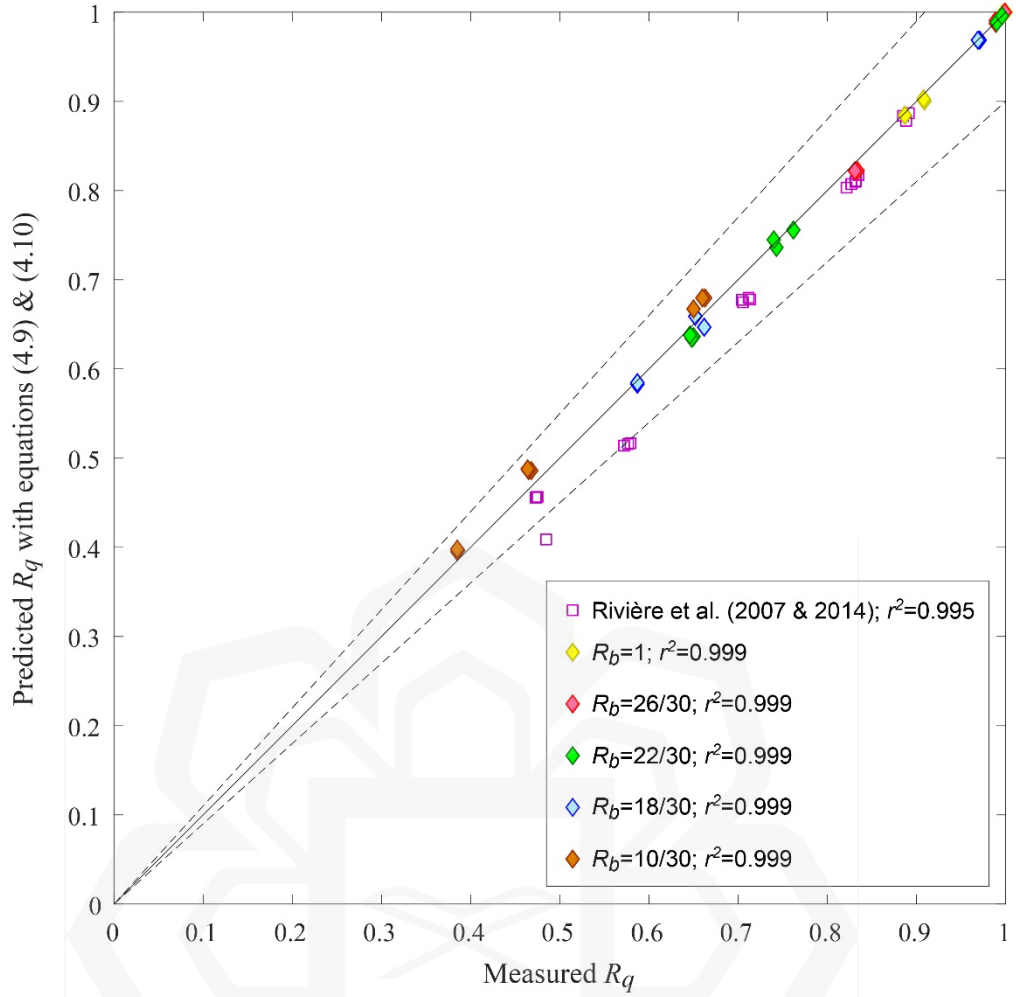


Figure 4.11 Evaluation of equations (4.9) and (4.10) using present data and from literature in free-recirculation transcritical regime.

In addition for another bifurcation angle (75°), Lakshmana Rao & Sridharan, (1967) proposed a 3rd order polynomial adjustment to obtain a correlation for K and for α . Applying the same approach to the authors' 90° -angle dataset generates the following equation:

$$\begin{aligned} K &= 0.43R_b^3 - 0.48R_b^2 - 0.35R_b + 1.51 \\ \alpha &= 0.95R_b^3 - 2.30R_b^2 + 2.06R_b + 0.29 \end{aligned} \quad (4.10)$$

Figure 4.11 validates the applicability of equations (4.9) and (4.10) in predicting the discharge distribution of the present free-recirculation transcritical flows with R_b values other than those used by the authors (Lakshmana Rao & Sridharan, 1967)].

4.3.4 Discharge distribution – all flow configurations

As affirmed above, the criterion $F_{yo} < 0.3$ first considered in §4.3.2 ensures that the flow configuration is in subcritical regime everywhere (without a critical section at the *vena contracta*). However, as expressed by Rivière et al. (2014), “Law & Reynolds, (1966) found that the critical Froude number for flow control was approximately $F_{yo} = 0.7$, which is coherent with our observations, but Ramamurthy & Satish, (1988), using the data of Lakshmana Rao et al. (1968) proposed a value of 0.35 for this critical Froude number”. Rivière et al. (2014) proposed a limit of $F_{yo} = 0.65$; however, the authors indicated it depends on the other boundary conditions, namely, C_x (or h_{xo}) at the outlet of the main branch, and F_{bx} at the inlet. The measured outlet discharges and water depths, as performed herein, do not permit a clear delimitation of subcritical and drowned-recirculation transcritical regimes, as both exhibit downstream control by the outlet weir of the side branch. Conversely, graphs, such as those in Figure 4.10, allow an identification of the limit in transcritical flows between drowned recirculation (with a critical section in the *vena contracta* but with a slight downstream control by the side branch outlet) and free-recirculation (without downstream control from the side branch). This limit (dashed lines in Figure 4.10) appears to increase with decreasing R_b for low F_{bx} values (i.e., from $F_{yo} = 0.35$ to 0.6 for $F_{bx} = 0.019$ as R_b decreases from 1 to 1/3), as well as with increasing F_{bx} (or Q_{xi}). Thus, a dedicated analysis is required to derive a definitive correlation to limit F_{yo} as a function of F_{bx} and R_b .

In practice for T-bifurcation flows, the regime of subcritical, drowned-recirculation transcritical, or free-recirculation transcritical is typically not known beforehand. Nevertheless, a simple methodology was proposed by Rivière et al. (2007) to predict the discharge distribution for any flow condition by advocating to maintain the minimum R_q value predicted by the correlations available in subcritical and free-recirculation transcritical regimes. This method is replicated as follows:

$$R_q = \min [\text{equation (4.8), equations (4.9-4.10)}] \quad (4.11)$$

Figure 4.12 confirms that this approach is efficient in predicting the discharge distribution for any flow configuration (any boundary condition triplet of Q_{xi} , h_{xo} , h_{yo}). Note that this excludes the case of a supercritical flow in the upstream branch, previously studied by Rivière & Perkins (2004), which can occur only in a steep channel or downstream from a hydraulic structure, such as a sluice gate or a weir. Note also that the validity of equation (4.11) can be affected by the location of the measurement of the downstream water depths (h_{xo} , h_{yo}) if this occurs far downstream from the intersection. One-dimensional backwater computations would then have to be applied to relate the downstream water depths to the water depths in the vicinity of the intersection at both outlets.



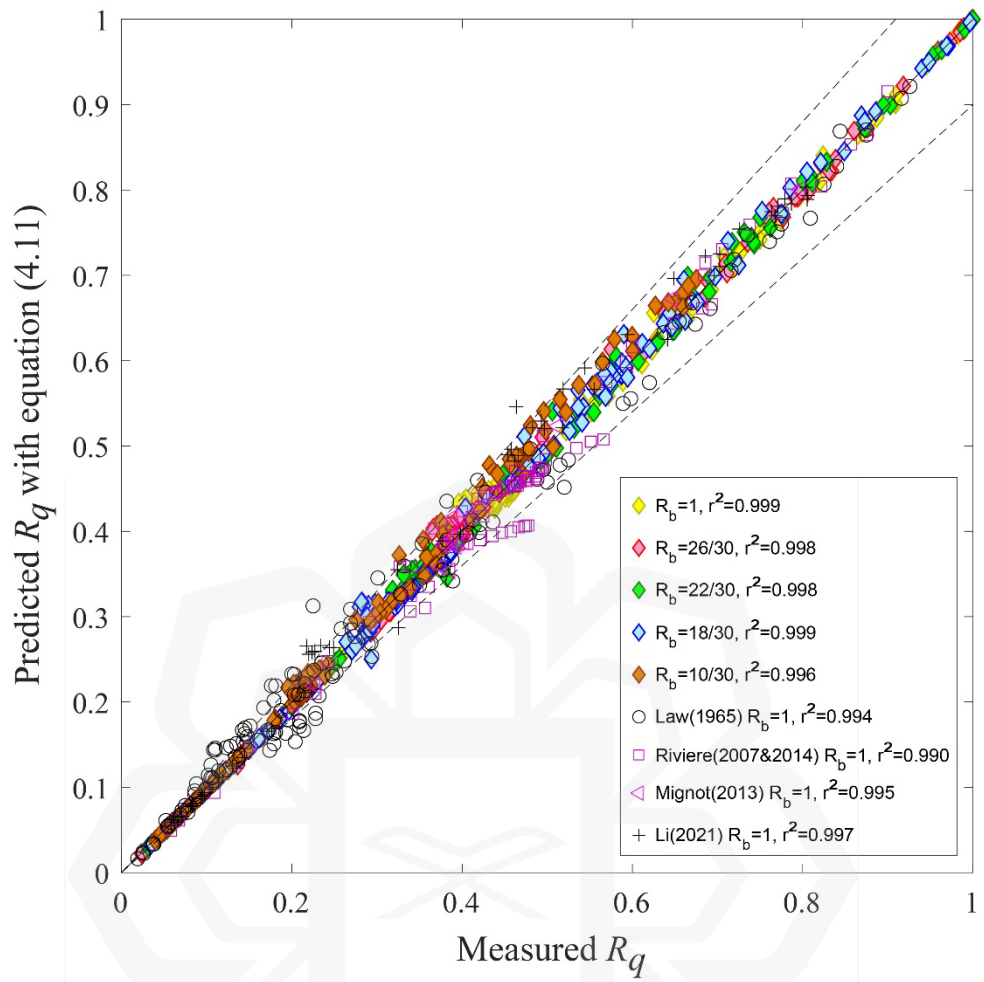


Figure 4.12 Predicted discharge distribution with equation (4.11) for all 967 tested flow configurations (299 from the literature and 668 from present investigation).

4.3.5 Discharge distribution – alternative approach based on measured water depths

The method proposed in equation (4.11) to predict the discharge distribution in subcritical equation (4.8) and transcritical [equations (4.9) and (4.10)] regimes is based on non-dimensional parameters resulting from the dimensional analysis provided in equation (3.5), namely, the Froude numbers in the downstream branches (F_{xo} and F_{yo}) and the branch width ratio R_b . However, both equations (4.8) and (4.9) are implicit, and must be solved iteratively, as Froude numbers are computed using the discharges in the downstream branches (Q_{xo} and Q_{yo}), themselves determined by the discharge distribution R_q . Considering the definitions of the downstream Froude numbers [equation (3.5)], equations (4.8) and (4.9) can alternatively be expressed as the following, respectively:

$$R_q = a \ln \left[\frac{b[\exp(1/a)-1]R_q}{[\exp(1/a)-1](1-R_q)\left(\frac{h_{yo}}{h_{xo}}\right)^{3/2} + bR_q} + 1 \right], \quad (4.12)$$

$$a = 1.012 \text{ and } b = 0.941$$

$$R_q = 10^{-K \left[\frac{Q_{xi}(1-R_q)}{b_x h_{xo}^{3/2} g^{1/2}} \right]^\alpha} \quad (4.13)$$

with α and K given by equation (4.10).

As such, the discharge distribution in equation (4.12) or (4.13) can be estimated based solely on the measured boundary conditions, that is, the upstream discharge Q_{xi} and downstream water depths h_{xo} and h_{yo} .

Nevertheless, Figure 4.13(a) shows that for subcritical flows, R_q in equation (4.12) is sensitive to $(h_{yo}/h_{xo})^{3/2}$ experimental uncertainties. This can also be proved analytically using the following approximation of equation (4.12) [see Figure 4.13(a)]:

$$R_q \approx -13.6(h_{yo}/h_{xo})^{3/2} + 13,$$

$$\text{such that } dR_q \approx \frac{3}{2} * 13.6(h_{yo}/h_{xo})^{3/2} (dh/h_{yo} + dh/h_{xo}).$$

$$\text{Considering } h_{xo} \approx h_{yo}, dR_q \approx 3 * 13.6(dh/h) \approx 40 dh/h.$$

Assuming $dh/h = 1\%$ ($dh = 0.5$ mm and $h \approx 50$ mm) this yields $dR_q \approx 0.4$, a substantial value of uncertainty, thus explaining the large discrepancies between the measured data and equation (4.12), as shown in Figure 4.13(a). Note that equation (4.12) indicates that the subcritical regime will occur only if the side-to-downstream branch ratio of water depths, h_{yo}/h_{xo} , is in the range of 0.917 to 0.968.

Conversely, for bifurcated flow in transcritical conditions, Figure 4.13(b) shows that equation (4.13) estimates the discharge distribution fairly well based on the measured inflow discharge Q_{xi} and outlet water depth in the main branch h_{xo} .

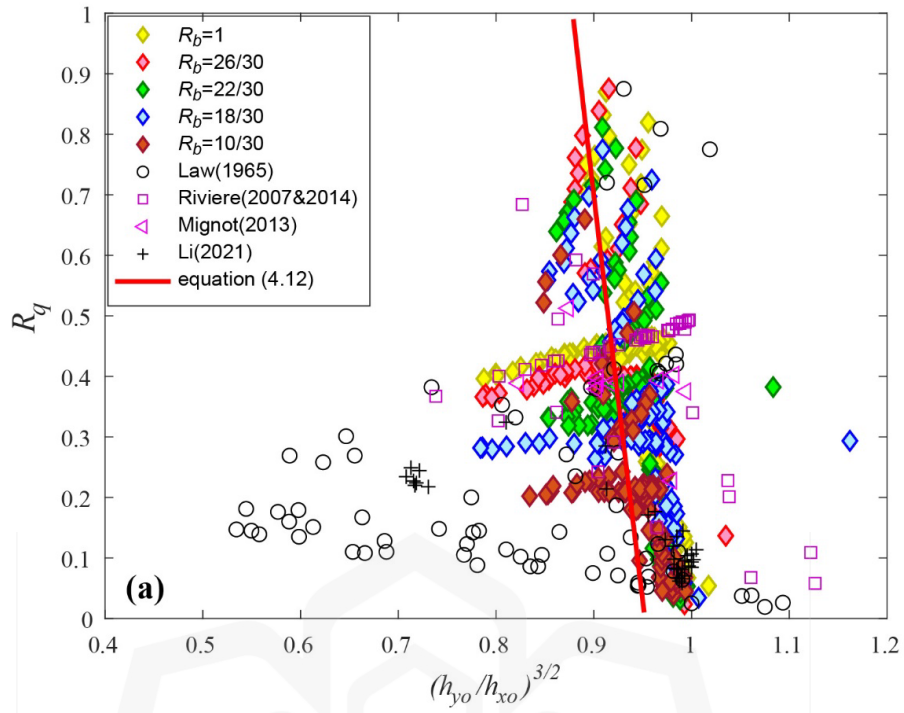


Figure 4.13(a) Effect of $(h_{yo}/h_{xo})^{3/2}$ on the measured discharge distribution (symbols) also compared with equation (4.12) (red line).

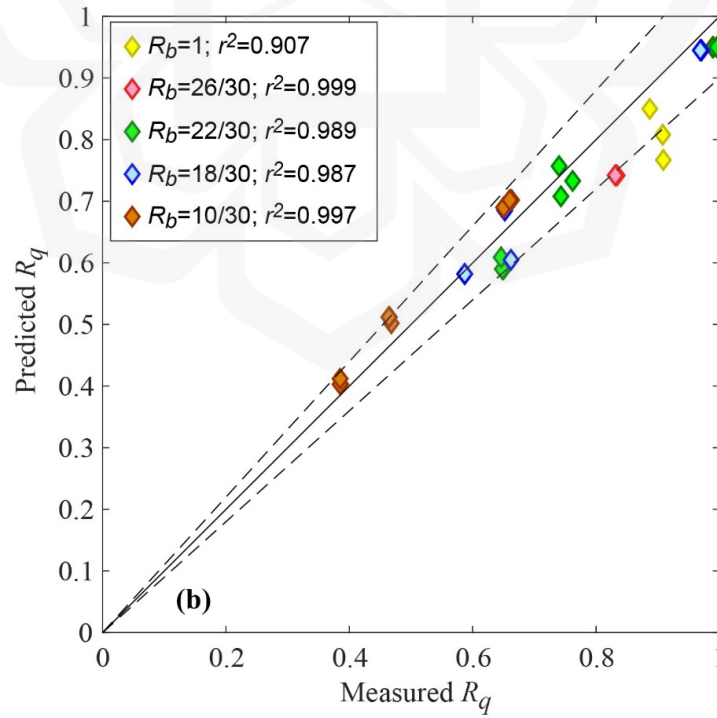


Figure 4.13(b) Comparison of measured and predicted equation (4.13) for discharge distributions in free-recirculation transcritical flow regime.

4.3.6 Discharge distribution – predictive approach

In most occasions in the field, the downstream boundary conditions (at the outlets of both branches) correspond to previously derived stage discharge relationships similar to equation (4.2) [developed by Rivière et al. (2007) employing correlation fitting techniques] rather than measured or fixed water depths as in reservoirs. Utilizing the latest experimental dataset (Appendix B), these have been updated to the following:

$$h_{xo} = \left(\frac{Q_{xo}}{0.485b_x\sqrt{2g}} \right)^{2/3} + C_x \dots\dots\dots (4.14a)$$

similarly, for lateral or branch channel

$$h_{yo} = \left(\frac{Q_{yo}}{0.471b_y\sqrt{2g}} \right)^{2/3} + C_y \dots\dots\dots (4.14b)$$

Upon plotting equation (4.14) in Figures 4.14 and 4.15, the agreement with measured data shows improvement with coefficients of determination (r^2) ranging between 0.9982 to 0.9997 as compared to Figures 4.3 and 4.4 for equation (4.2). Reinforced by visual observation of such an exceptional fit, it is resolved that the new stage discharge (or rating curve) relationships for open-channel bifurcations have now been established for this research, and can be adopted for practical applications with confidence.

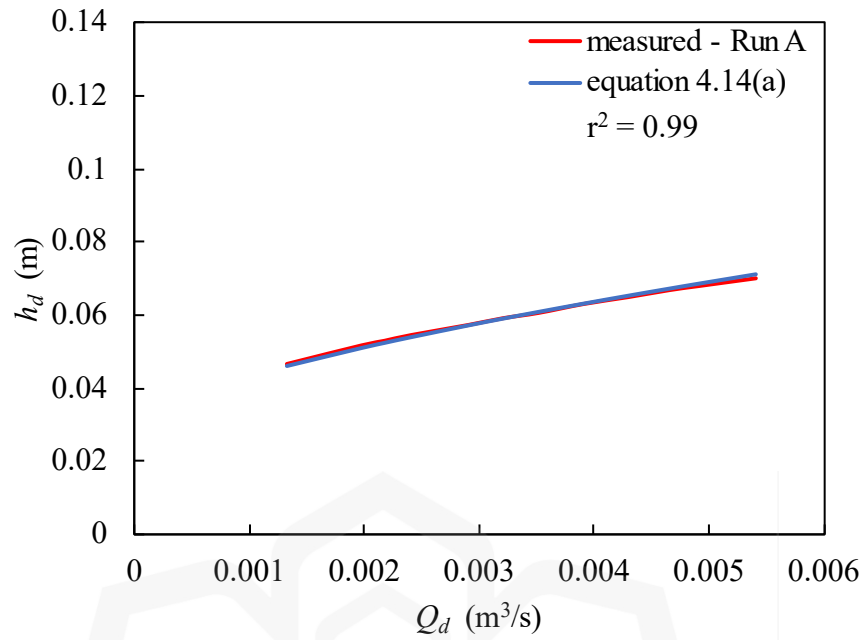


Figure 4.14(a) Downstream stage-discharge plots for measured and computed flow depths [equation 4.14(a)] – run A in Table 3.2 ($C_x = C_y = 30$ mm)

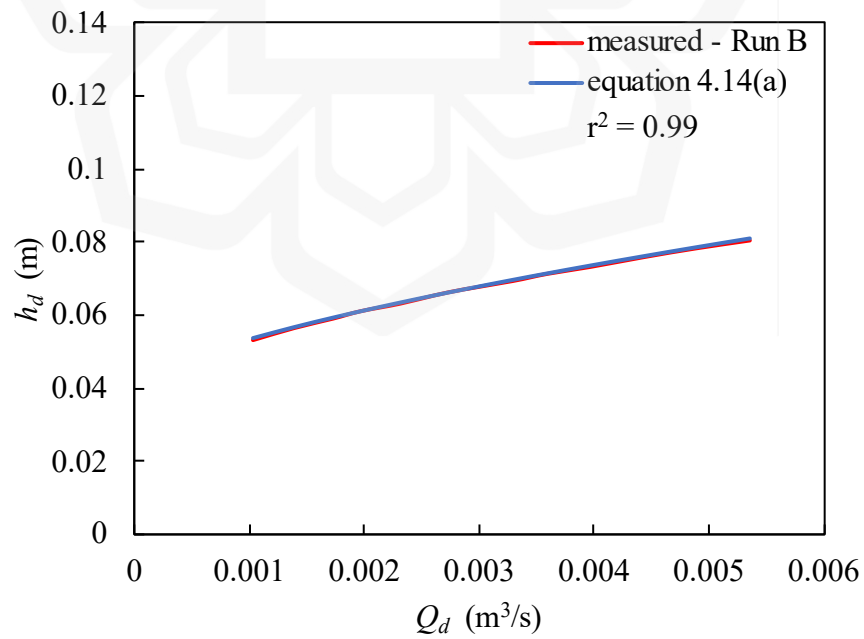


Figure 4.14(b) Downstream stage-discharge plots for measured and computed flow depths [equation 4.14(a)] – run B in Table 3.2 ($C_x = C_y = 40$ mm)

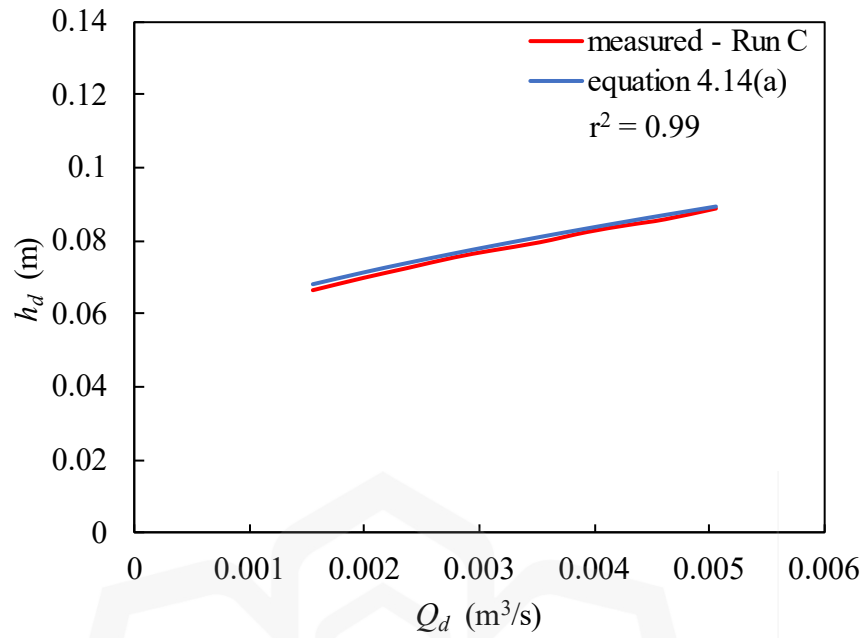


Figure 4.14(c) Downstream stage-discharge plots for measured and computed flow depths [equation 4.14(a)] – run C in Table 3.2 ($C_x = C_y = 50$ mm)

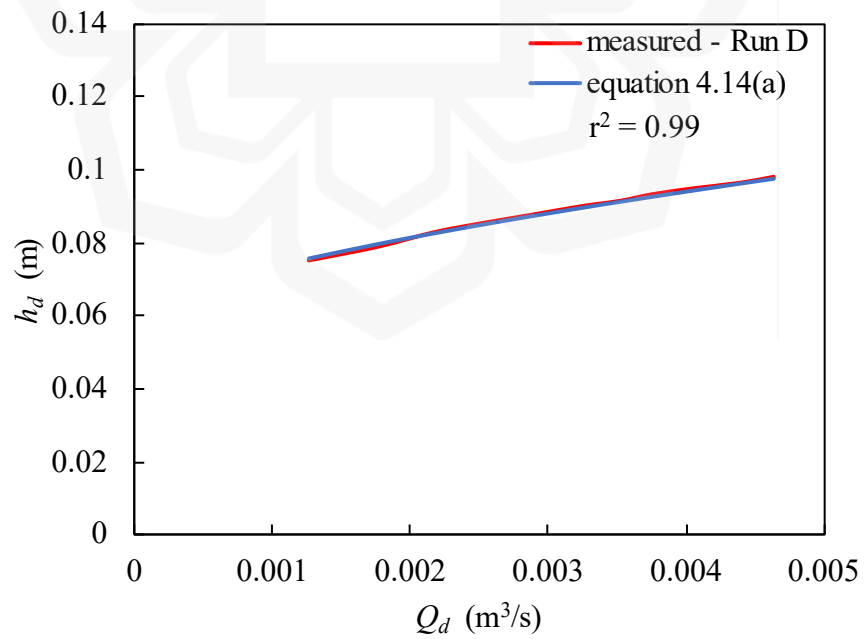


Figure 4.14(d) Downstream stage-discharge plots for measured and computed flow depths [equation 4.14(a)] – run D in Table 3.2 ($C_x = C_y = 60$ mm)

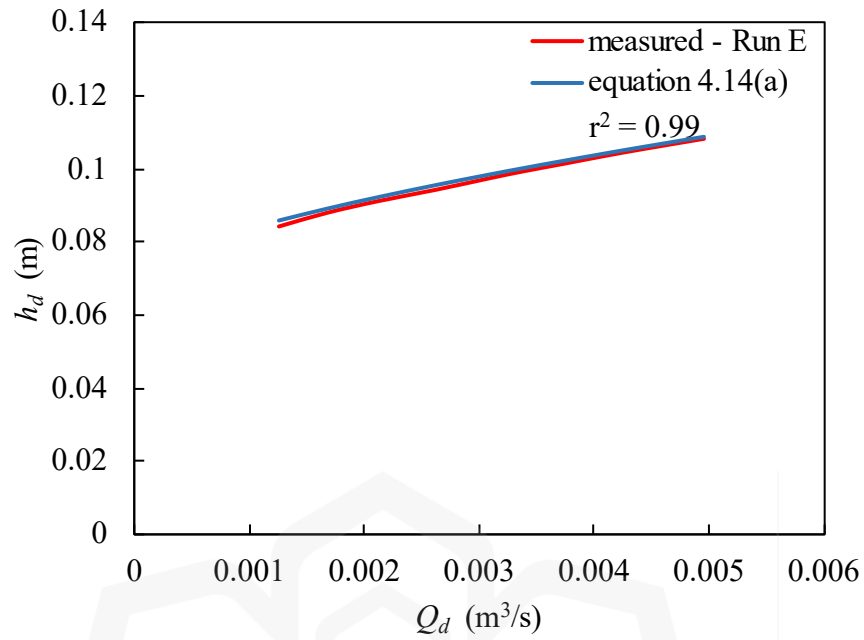


Figure 4.14(e) Downstream stage-discharge plots for measured and computed flow depths [equation 4.14(a)] – run E in Table 3.2 ($C_x = C_y = 70$ mm)

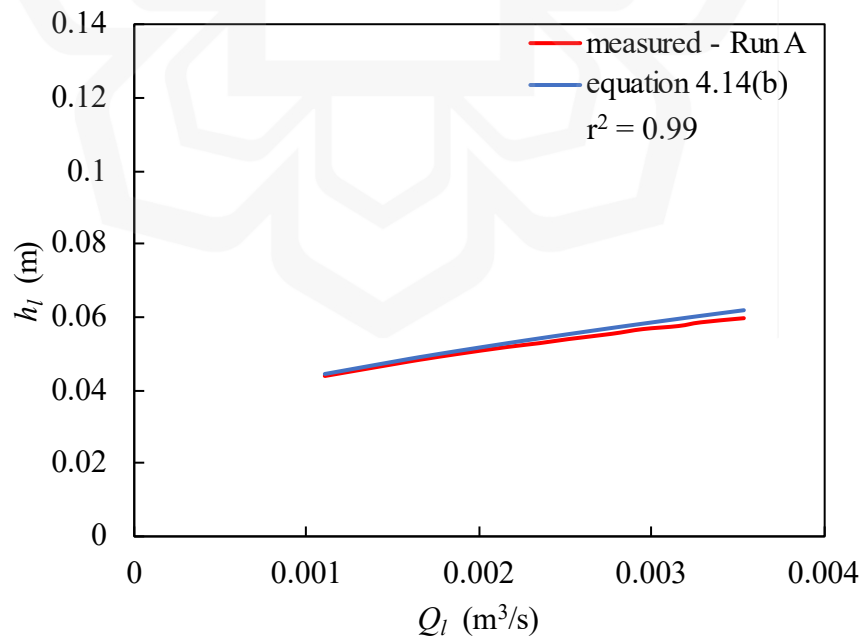


Figure 4.15(a) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.14(b)] – run A in Table 3.2 ($C_x = C_y = 30$ mm)

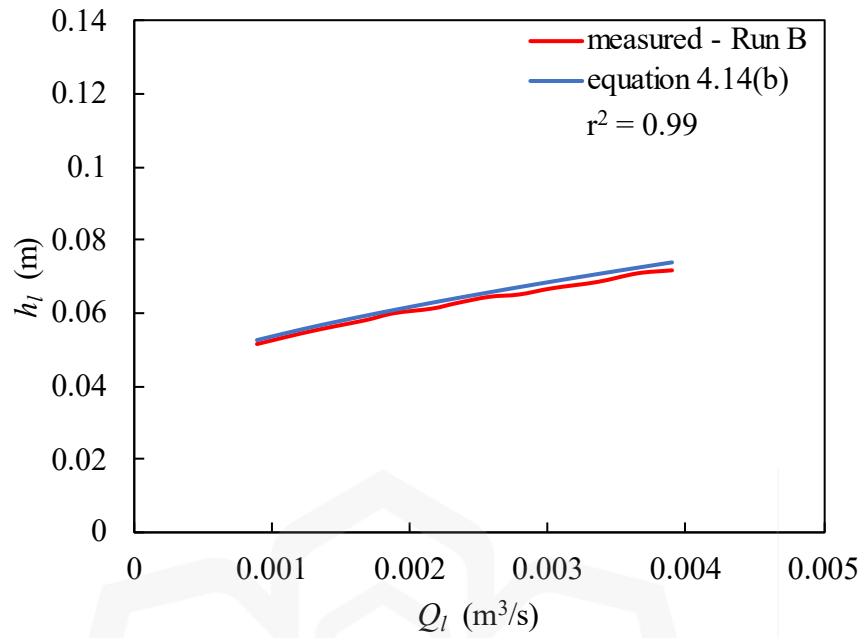


Figure 4.15(b) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.14(b)] – run B in Table 3.2 ($C_x = C_y = 40$ mm)

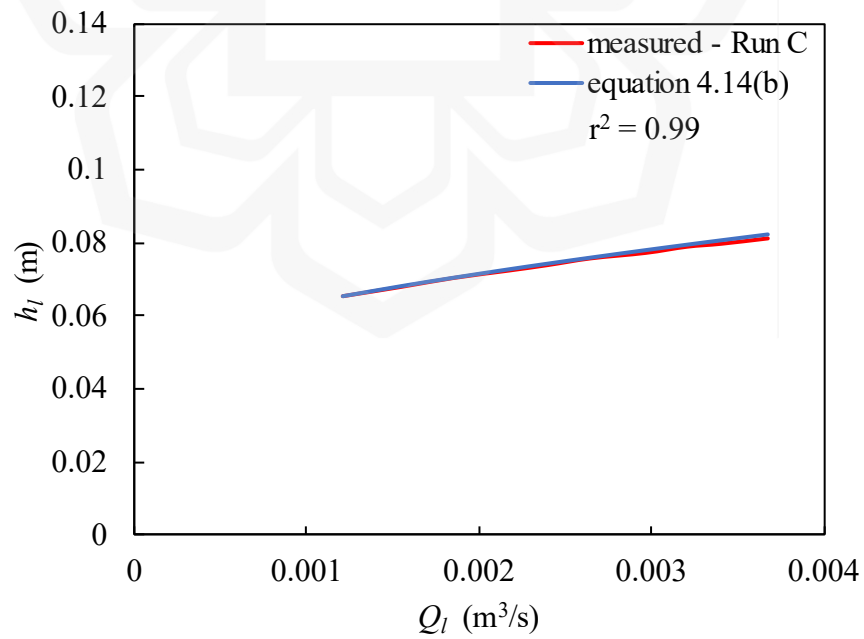


Figure 4.15(c) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.14(b)] – run C in Table 3.2 ($C_x = C_y = 50$ mm)

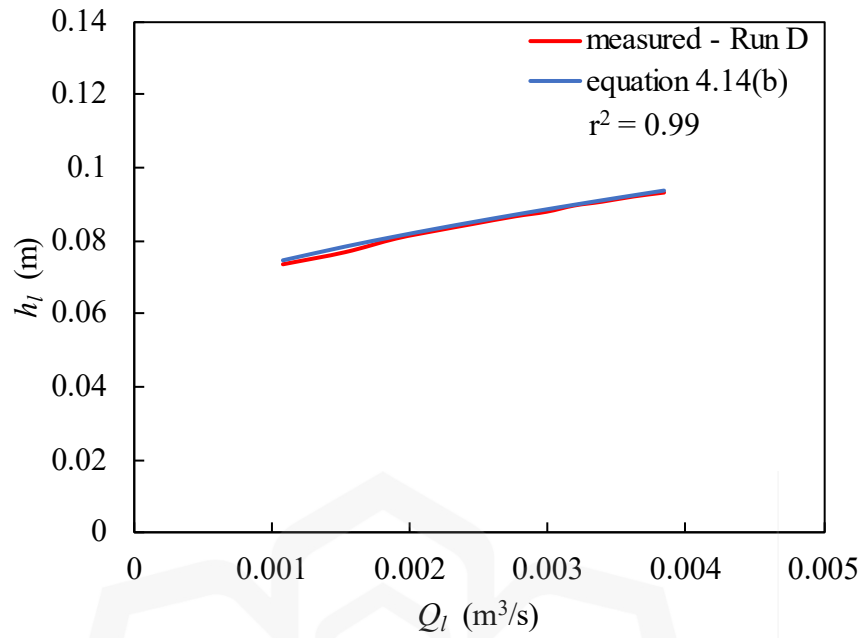


Figure 4.15(d) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.14(b)] – run D in Table 3.2 ($C_x = C_y = 60$ mm)

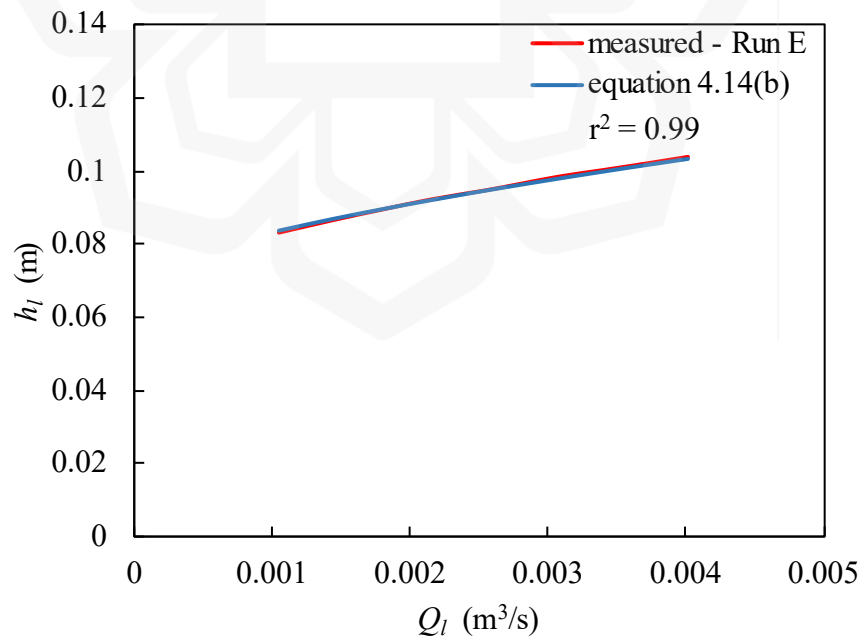


Figure 4.15(e) Lateral/branch stage-discharge plots for measured and computed flow depths [equation 4.14(b)] – run E in Table 3.2 ($C_x = C_y = 70$ mm)

For subcritical conditions, equation (4.8) or (4.12), along with the two downstream stage discharge relationships [equation (4.14)], form a set of three equations for three unknown variables: R_q , F_{xo} , and F_{yo} in equation (4.8) (or R_q , h_{xo} , and h_{yo} in equation (4.12)). Similarly in free-recirculation transcritical flow regime, equations (4.9) and (4.10) [or equation (4.13) and (4.10) along with the downstream stage discharge relationship (equation (4.14a)) in the main branch], form a set of two equations for two unknown variables: R_q and F_{xo} in equations (4.9) [or R_q and h_{xo} in equation (4.12)]. Thus, these sets of equations predict the discharge distribution in the bifurcation without any measurement in the downstream branches, knowing only the inlet discharge Q_{xi} , downstream stage discharge relationships, and bifurcation geometry (R_b notably).

The approach used in equation (4.11) [established by Rivière et al. (2014)] is then applied for a particular Q_{xi} and R_b as follows:

1. Firstly, the subcritical discharge distribution R_q is obtained by the set of three equations, namely (4.8) or (4.12) coupled with the two downstream stage discharge relationships in (4.14).
2. Secondly, R_q in free-recirculation transcritical conditions is computed by the three respective equations of (4.9) and (4.10) or (4.9) and (4.10), coupled with the downstream main branch stage-discharge relationship (4.14a).
3. The smaller of these two values then forms the predicted R_q without the need to determine the flow regime beforehand.

The comparison between the predicted and experimental values is limited to the present experiments only, as this test requires stage discharge relationships that are not available in the literature. The predicted discharge distribution is of high quality, with an average error limited to $dR_q = 0.029$ compared to the measurements, as plotted in Figure 4.16. Based on available literature and present knowledge, this strategy is the most adaptable operational method for predicting the discharge distribution in a horizontal or limited-slope T-bifurcation, both for subcritical and transcritical regimes, preventing a supercritical flow regime in the main branch.

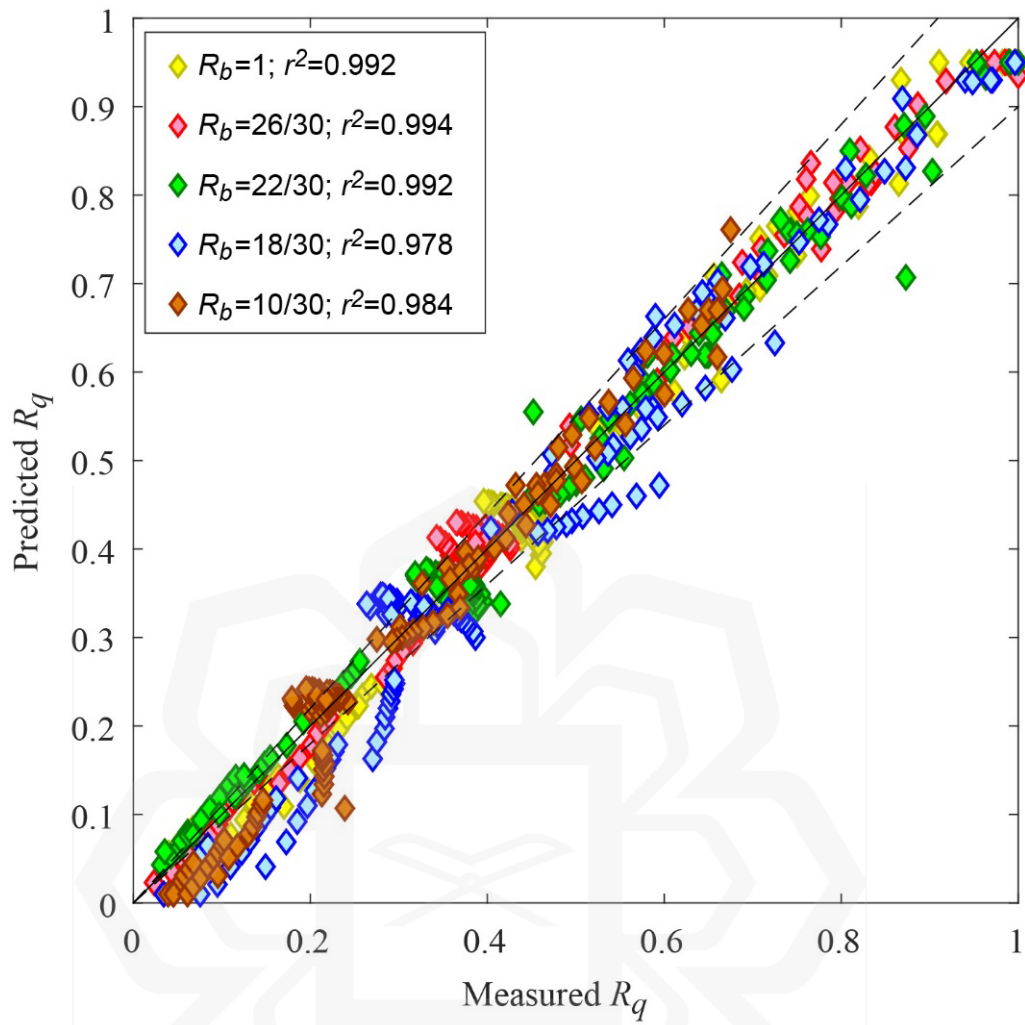


Figure 4.16 Evaluation of the predicted discharge distribution using the minimum between equation (4.8) [or (4.12)] and equation (4.9) [or (4.13)] coupled with downstream stage discharge relationships for a known upstream discharge Q_{xi} and a particular R_b value.

4.3.7 Summary of discharge distribution relationships

The present work aims to summarize, validate, and extend the available discharge distribution correlations for horizontal T-shaped bifurcations with simplified geometries in which the side branch is narrower or of the same width as the main channel. The two regime conditions in the lateral or branch channel are addressed with overall subcritical flow in the main channel. Firstly, the new empirical correlation for bifurcation in the fully subcritical regime [equation (4.8)], compared to previous correlations from the literature (Table 2.3), results in substantial accuracy improvement and can be used regardless of the downstream conditions of the bifurcation (mainly with fixed water depths or stage discharge relationships). Secondly, for flows in the free-recirculation transcritical regime, the correlation proposed by Lakshmana Rao & Sridharan, (1967) [equations (4.9) and (4.10)] agrees well with the present experimental datasets and those available in the literature. Equation (4.11) is then applied for any flow condition without the need to determine its regime in the bifurcation beforehand. In Table 2.3, it can be discerned that all preceding work derived relationships were mainly applicable to either subcritical or supercritical conditions in the side/lateral branch.

Next, to circumvent the implicitness of equations (4.8) & (4.9), which are solved iteratively as they depend on F_{xo} (Q_{xo}), F_{yo} (Q_{yo}) and R_b , the discharge distribution in equation (4.12) or (4.13) are derived and proposed as they can be estimated based solely on the measured boundary conditions, that is, the upstream discharge Q_{xi} and downstream water depths h_{xo} and h_{yo} .

Finally, a general predictive method is developed for the discharge distribution based on the known upstream discharge plus branch width ratio and stage discharge (rating curve) relationships in the downstream branches. Such an approach is then applicable out in the field or real-world situations where the downstream discharges in both channel reaches correspond to ‘rating curves’ already established via gauging procedures. All plots of predicted R_q exhibit very good fit and agreement with the experimental data of previous investigators and this research.

Nevertheless, this investigative approach has several constraints. Initially, the side/lateral branch remained slightly wide, with a minimum width ratio $R_b = 0.33$, whereas Lakshmana Rao & Sridharan, (1967) considered at least $R_b = 0.25$, and field conditions can exhibit significantly narrower side branches. Thus, upcoming work can be dedicated to bifurcations with significantly narrower side branches. Moreover, the present configuration is restricted to right-angle or T-shape bifurcations, and although Lakshmana Rao & Sridharan, (1967) extended their work to four additional angles, no correlation is, to the best of found knowledge, available for subcritical flows in bifurcations with an angle departing from 90° . This can also constitute valuable future research.



CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

In this last chapter, the findings and conclusions of this research in addressing the stipulated objectives (Chapter 1) are presented and summarized. At the end, the residual questions to be extended to future studies as well possible further work are then highlighted and recommended.

5.1 CONCLUSIONS

For the first objective, i.e to determine experimental techniques of visualizing flow structures and distribution in bifurcated open-channel flow, the ink-injected technique adopted by Momplot et al. (2017) to visually distinguish the 2D standard recirculation and 3D helical flow structures in the bifurcation was assessed during the early trial runs in the flume. Based on video captures, both manual and tank-assisted procedures were determined (in §3.5) to be inadequate and impractical due to unavoidable transients as well as instabilities in the experiments. Therefore, it was determined at the early stages of the research to test more reliable techniques of flow visualization such as tracking movement of buoyant chips or a long plastic strip/thread of visible colour. However, it was observed that floating particles (paraffin and wood chips) were only useful for patterns on the water surface and was not definitive. Thus, as the utilization a long plastic strip provided promising results, it is decided to adopt this method for *visually* identifying such flow structures and patterns subject to further investigation and confirmation.

Continuing from above, the second objective is then to investigate (other, non-visual) methods to identify three-dimensional flow structures in bifurcated open-channel flow. Based on the practice of Momplot et al. (2017), $(F_{bx})^2$ values were plotted versus b_x/h_u , and the resulting Figure 4.1 exhibited a noticeable difference in slopes or curvature of the data points which apparently differentiated the two distinct flow patterns (2D-closed and 3D-helical). The steeper curves seem to indicate more complex structures of 3D-helical recirculation occurring at higher discharges (deeper

depths) and corresponding Froude numbers. This proved a useful approach in deriving formulations like equation (2.1) once visualization techniques have been established.

Furthermore, upon plotting the loss of fluid or hydraulic power after the bifurcation with respect to upstream (total) discharge Q_{in} , in Figure 4.2, the observed trend is most likely caused by the higher rate of energy reduction as the flow splits to the downstream areas with the increase in discharge (and Froude no.). It is evident that the standard 2D closed patterns initially occurring in the recirculation zone are ‘converted’ to more complex 3D helixes, the latter apparently generated by more power. This indicates the formation of vortices and helixes that require more energy, similar to the energy-cascade concept in turbulent flow, thus resulting in the loss of fluid power beyond a certain transformation point. Therefore, the power loss % at point of conversion can be concluded as the possible threshold to distinguish between the two distinct structures in bifurcated open-channel flow.

In this research, the experimental investigations and ensuing results analysis were conducted to address the following objectives 3 and 4 in combination:

3. To improve empirical flow distribution relationships in open channel T-shape bifurcations
4. To establish the effect of reduced side branch width on empirical flow distribution relationships in open channel T-shape bifurcations

In this regard, the conclusions of the work are:

- formulation of a new empirical correlation, equation (4.8), with substantial accuracy improvement for bifurcated flow in the fully subcritical regime – this can be used regardless of the downstream conditions
- adoption of equations (4.9) and (4.10) proposed by Lakshmana Rao & Sridharan, (1967) for flows in the free-recirculation transcritical regime
- application of equation (4.11) for any flow condition without the need to determine its regime in the bifurcation beforehand

- to circumvent the implicitness of equations (4.8) & (4.9), equations (4.12) and (4.13) are derived, which can be estimated based solely on the measured boundary conditions, namely upstream discharge and downstream water depths
- a general predictive method, with equation (4.14) as an example, is developed for bifurcated flow distribution based on the known upstream discharge plus width ratio and stage-discharge relationships in the downstream branches – this is practically applicable in the field or real situations where the downstream discharges in both channel reaches correspond to already established ‘rating curves’

To confirm the novelty of this study, all derived equations and the established procedure listed above are compared with previous work summarized in Tables 2.2 and 2.3. As can be discerned, essentially none of the preceding investigations covered concurrently both subcritical and transcritical flow conditions in the lateral/side channel. Secondly, a majority of past efforts were mainly focused on equal width (lateral/side to main) ratios. Moreover, all previously derived empirical correlations or equations for bifurcated discharge distributions are implicit as functions of Froude numbers among other parameters, while some require iterative procedures to solve. All these ‘concerns’ have been addressed and dealt with in the present research.

5.2 RECOMMENDATIONS FOR FURTHER STUDIES

The investigations utilizing mainly experimental methods completed in this research faced several challenges and limitations. These are summarized and supplemented with recommendations for further studies as follows.

As stated above, the manual by-eye identification of flow structure was hindered by a lack of a suitable tested technique and available material in the mainly steady flow conditions, but with inherent transients and instabilities, in the flume. Thus, the first recommendation for upcoming work is to establish reliable and consistent procedure to visually classify the two distinct 2D or 3D flow patterns in the recirculation zone, supported by more sophisticated (costly) indirect methods such as Particle Image

Velocimetry (PIV), Particle Tracking Velocimetry (PTV) and Large-scale Particle Image Velocimetry (LS-PIV). A flow-structure distinguishing formula, such as equation (2.1) and plotted in Figure 2.9 (Momplot et al., 2017) , can then be effectively developed, tested and subsequently adopted in the future.

Secondly, although Lakshmana Rao & Sridharan, (1967) considered a minimum ratio $R_b = 0.25$, the side/lateral branch width ratios were no less than 0.33. Furthermore, real conditions in the field more often than not exhibit much narrower bifurcations, especially for lateral raw water supply intakes. As such, further analyses specifically for, but not limited to, R_b values less than 0.3 are recommended.

Thirdly, the discharge distribution (R_q) correlations have been derived for a side/lateral channel perpendicular to the main flow, defined as a 90°- or T-bifurcation. Apart for the work of Lakshmana Rao & Sridharan, (1967) decades ago which investigated four additional angles, no R_q formulation is found thus far for subcritical flows in non T-bifurcations. Therefore, all angles besides 90° should be seriously considered in future research to extend and improve discharge distribution relationships for bifurcated open-channel flow.

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APPENDIX A: Completed experimental runs

R_b	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)
1	2.42	3	3	2.76	5	5	6.64	7	4
	3.56	3	3	3.55	5	5	7.22	7	4
	3.86	3	3	4.22	5	5	7.83	7	4
	4.27	3	3	4.95	5	5	2.98	7	5
	5.06	3	3	5.50	5	5	3.55	7	5
	5.55	3	3	6.17	5	5	4.10	7	5
	6.03	3	3	6.90	5	5	4.76	7	5
	6.67	3	3	7.50	5	5	5.35	7	5
	7.24	3	3	8.02	5	5	6	7	5
	7.81	3	3	8.73	5	5	6.52	7	5
	8.20	3	3	3.92	5	7	7.10	7	5
	8.94	3	3	4.56	5	7	7.61	7	5
	7.78	3	7	5.05	5	7	3	7	6
	8.34	3	7	5.66	5	7	3.63	7	6
	8.90	3	7	6.28	5	7	4.24	7	6
	1.92	4	4	6.81	5	7	4.90	7	6
	2.60	4	4	7.43	5	7	5.55	7	6
	3.19	4	4	8.04	5	7	6.11	7	6
	3.65	4	4	8.67	5	7	6.76	7	6
	4.19	4	4	2.35	6	6	7.35	7	6
	4.77	4	4	3.30	6	6	7.88	7	6
	5.20	4	4	4.08	6	6	2.31	7	7
	5.86	4	4	4.73	6	6	3.12	7	7
	6.38	4	4	5.31	6	6	3.91	7	7
	6.99	4	4	6	6	6	4.77	7	7
	7.79	4	4	6.52	6	6	5.62	7	7
	8.53	4	4	6.95	6	6	6.17	7	7
	9.25	4	4	7.46	6	6	6.81	7	7
	5.14	4	7	8.02	6	6	7.48	7	7
	5.76	4	7	8.48	6	6	8.08	7	7
	6.28	4	7	3.20	6	7	8.96	7	7
	6.84	4	7	3.81	6	7			
	7.43	4	7	4.40	6	7			
	8.04	4	7	5.06	6	7			
	3	5	0	5.67	6	7			
	6	5	0	6.25	6	7			
	3	5	1	6.85	6	7			
	6	5	1	7.47	6	7			
	3	5	1	8.04	6	7			
	6	5	1	2.75	7	3			
	3	5	2	3.29	7	3			
	6	5	2	3.85	7	3			
	3	5	2	4.51	7	3			
	6	5	2	4.97	7	3			
	9	5	2	5.60	7	3			
	3	5	3	3.04	7	4			
	6	5	3	3.77	7	4			
	9	5	3	4.34	7	4			
	3	5	4	4.92	7	4			
	6	5	4	5.53	7	4			
	9	5	4	6.06	7	4			

R_b	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)
26/30	2.64	3	3	2.36	5	5	7.04	7	4
	3.64	3	3	2.88	5	5	7.69	7	4
	4.26	3	3	3.36	5	5	3.30	7	4
	4.85	3	3	3.98	5	5	3.97	7	4
	5.47	3	3	4.54	5	5	4.58	7	4
	5.95	3	3	5.48	5	5	5.24	7	4
	6.61	3	3	6.04	5	5	5.85	7	4
	7.02	3	3	6.49	5	5	6.46	7	4
	7.54	3	3	6.99	5	5	7.04	7	5
	7.92	3	3	7.47	5	5	7.55	7	5
	6.68	3	7	4.67	5	7	7.98	7	5
	7.17	3	7	5.09	5	7	2.49	7	7
	7.69	3	7	5.53	5	7	3.13	7	7
	8.15	3	7	6.07	5	7	3.68	7	7
	8.46	3	7	6.59	5	7	4.35	7	7
	2.63	4	4	7.05	5	7	4.90	7	7
	3.13	4	4	7.45	5	7	5.39	7	7
	3.81	4	4	2.80	6	6	5.88	7	7
	4.53	4	4	3.38	6	6	6.41	7	7
	5.05	4	4	3.77	6	6	6.89	7	7
	5.73	4	4	4.38	6	6	7.58	7	7
	6.25	4	4	4.91	6	6			
	6.83	4	4	5.49	6	6			
	7.43	4	4	6	6	6			
	7.97	4	4	6.59	6	6			
	5.27	4	7	7.12	6	6			
	5.69	4	7	7.54	6	6			
	6.01	4	7	4.50	6	7			
	6.30	4	7	4.92	6	7			
	6.71	4	7	5.33	6	7			
	6.96	4	7	6	6	7			
	7.59	4	7	6.52	6	7			
	8.13	4	7	7	6	7			
	8.45	4	7	7.72	6	7			
	3	5	0	8.11	6	7			
	6	5	0	8.40	6	7			
	3	5	1	8.91	6	7			
	6	5	1	3.13	7	3			
	3	5	1	3.76	7	3			
	6	5	1	4.41	7	3			
	3	5	2	5	7	3			
	6	5	2	5.58	7	3			
	3	5	2	6.16	7	3			
	6	5	2	6.84	7	3			
	9	5	2	7.32	7	3			
	3	5	3	7.93	7	3			
	6	5	3	2.92	7	4			
	9	5	3	4.09	7	4			
	3	5	4	4.63	7	4			
	6	5	4	5.28	7	4			
	9	5	4	6.47	7	4			

R_b	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)
22/30	2.77	3	3	2.52	5	5	3.18	7	5
	3.45	3	3	3.31	5	5	3.79	7	5
	4.07	3	3	4.12	5	5	4.39	7	5
	4.59	3	3	4.71	5	5	5.02	7	5
	5.25	3	3	5.41	5	5	5.60	7	5
	5.79	3	3	6.34	5	5	6.23	7	5
	6.46	3	3	7.16	5	5	6.81	7	5
	7.04	3	3	7.86	5	5	7.43	7	5
	7.64	3	3	8.48	5	5	8.07	7	5
	7.76	3	7	3.67	5	7	3.04	7	5
	8.21	3	7	4.23	5	7	3.68	7	5
	8.71	3	7	4.79	5	7	4.33	7	5
	9.10	3	7	5.35	5	7	4.95	7	5
	9.64	3	7	5.84	5	7	5.42	7	5
	2.34	4	4	6.18	5	7	6.23	7	5
	2.97	4	4	6.53	5	7	6.94	7	5
	3.67	4	4	2.66	6	6	7.57	7	5
	4.08	4	4	3.04	6	6	8.23	7	5
	4.45	4	4	4.38	6	6	2.52	7	7
	5.03	4	4	5.13	6	6	3.05	7	7
	5.61	4	4	5.83	6	6	3.52	7	7
	6.08	4	4	6.55	6	6	4.09	7	7
	6.72	4	4	7.20	6	6	4.49	7	7
	7.38	4	4	7.88	6	6	5.33	7	7
	5.67	4	7	8.45	6	6	6.04	7	7
	6.30	4	7	2.39	6	7	6.78	7	7
	7.05	4	7	3.07	6	7	7.37	7	7
	7.64	4	7	3.85	6	7	7.99	7	7
	8.24	4	7	4.50	6	7			
	9	4	7	5.20	6	7			
	9.59	4	7	5.94	6	7			
	3	5	0	6.55	6	7			
	6	5	0	7.19	6	7			
	9	5	0	8.01	6	7			
	3	5	1	3.16	7	3			
	6	5	1	3.78	7	3			
	9	5	1	4.42	7	3			
	3	5	1	5.08	7	3			
	6	5	1	5.64	7	3			
	9	5	1	6.23	7	3			
	3	5	2	6.88	7	3			
	9	5	2	7.39	7	3			
	3	5	2	8.06	7	3			
	6	5	2	3.33	7	4			
	9	5	2	3.89	7	4			
	3	5	3	5.11	7	4			
	6	5	3	5.70	7	4			
	9	5	3	6.28	7	4			
	3	5	4	6.87	7	4			
	6	5	4	7.35	7	4			
	9	5	4	7.96	7	4			

R_b	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)
18/30	2.69	3	3	6	5	4	4.26	7	4
	3.42	3	3	9	5	4	4.86	7	4
	4.16	3	3	3.08	5	5	5.39	7	4
	4.84	3	3	3.85	5	5	5.87	7	4
	5.47	3	3	4.45	5	5	6.33	7	4
	6.31	3	3	5.07	5	5	6.87	7	4
	6.93	3	3	5.77	5	5	7.37	7	4
	7.68	3	3	6.47	5	5	8.09	7	4
	8.16	3	3	7.17	5	5	3.09	7	5
	8.59	3	3	7.84	5	5	3.71	7	5
	6.12	3	7	8.39	5	5	4.27	7	5
	6.71	3	7	3.02	5	7	4.87	7	5
	7.31	3	7	3.64	5	7	5.52	7	5
	8.02	3	7	4.25	5	7	6.13	7	5
	8.63	3	7	4.83	5	7	6.66	7	5
	9.21	3	7	5.48	5	7	7.32	7	5
	2.80	4	4	6.07	5	7	8.41	7	5
	3.48	4	4	6.62	5	7	2.86	7	6
	4.07	4	4	7.20	5	7	3.43	7	6
	4.82	4	4	7.79	5	7	4.03	7	6
	5.44	4	4	8.35	5	7	4.58	7	6
	6.12	4	4	2.58	6	6	5.12	7	6
	6.75	4	4	3.17	6	6	5.67	7	6
	7.29	4	4	3.72	6	6	6.21	7	6
	8.11	4	4	4.44	6	6	6.80	7	6
	4.00	4	7	5.04	6	6	7.37	7	6
	4.59	4	7	5.60	6	6	8.07	7	6
	5.25	4	7	6.94	6	6	2.69	7	7
	5.84	4	7	7.52	6	6	3.28	7	7
	6.41	4	7	8.16	6	6	3.92	7	7
	7.04	4	7	3.08	6	7	4.53	7	7
	7.65	4	7	3.67	6	7	5.10	7	7
	8.21	4	7	4.23	6	7	5.73	7	7
	8.82	4	7	4.85	6	7	6.31	7	7
	3	5	0	5.42	6	7	6.96	7	7
	6	5	0	5.99	6	7	7.51	7	7
	9	5	0	6.59	6	7	8.13	7	7
	3	5	1	7.17	6	7			
	9	5	1	7.74	6	7			
	3	5	1	8.17	6	7			
	6	5	1	2.71	7	3			
	9	5	1	3.29	7	3			
	3	5	2	4.02	7	3			
	9	5	2	4.56	7	3			
	3	5	2	5.20	7	3			
	6	5	2	5.77	7	3			
	9	5	2	6.26	7	3			
	3	5	3	6.95	7	3			
	6	5	3	7.57	7	3			
	9	5	3	2.78	7	4			
	3	5	4	3.42	7	4			

R_b	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)	Q_{xi} (l/s)	C_x (cm)	C_y (cm)
10/30	2.99	0.03	0.03	2.90	0.05	0.05	7.03	0.07	0.04
	3.64	0.03	0.03	3.68	0.05	0.05	7.62	0.07	0.04
	4.32	0.03	0.03	4.36	0.05	0.05	2.83	0.07	0.05
	5.14	0.03	0.03	4.98	0.05	0.05	3.43	0.07	0.05
	5.70	0.03	0.03	5.49	0.05	0.05	4.09	0.07	0.05
	6.20	0.03	0.03	5.94	0.05	0.05	4.67	0.07	0.05
	6.83	0.03	0.03	6.61	0.05	0.05	5.21	0.07	0.05
	7.66	0.03	0.03	7.14	0.05	0.05	5.80	0.07	0.05
	8.40	0.03	0.03	7.80	0.05	0.05	6.40	0.07	0.05
	5.86	0.03	0.07	3.16	0.05	0.07	7.02	0.07	0.05
	6.49	0.03	0.07	3.84	0.05	0.07	7.72	0.07	0.05
	7.09	0.03	0.07	4.40	0.05	0.07	3.09	0.07	0.06
	7.65	0.03	0.07	5.05	0.05	0.07	3.68	0.07	0.06
	8.22	0.03	0.07	5.60	0.05	0.07	4.38	0.07	0.06
	8.78	0.03	0.07	6.24	0.05	0.07	5.03	0.07	0.06
	9.41	0.03	0.07	6.64	0.05	0.07	5.65	0.07	0.06
	3.40	0.04	0.04	7.26	0.05	0.07	6.23	0.07	0.06
	3.99	0.04	0.04	7.85	0.05	0.07	7.41	0.07	0.06
	4.73	0.04	0.04	2.95	0.06	0.06	8.01	0.07	0.06
	5.84	0.04	0.04	3.78	0.06	0.06	2.87	0.07	0.07
	6.35	0.04	0.04	4.43	0.06	0.06	3.50	0.07	0.07
	6.98	0.04	0.04	5.16	0.06	0.06	4.09	0.07	0.07
	7.63	0.04	0.04	5.72	0.06	0.06	4.55	0.07	0.07
	3.70	0.04	0.07	6.26	0.06	0.06	5.21	0.07	0.07
	4.30	0.04	0.07	6.72	0.06	0.06	6.02	0.07	0.07
	4.88	0.04	0.07	7.32	0.06	0.06	6.64	0.07	0.07
	5.41	0.04	0.07	7.87	0.06	0.06	7.11	0.07	0.07
	6.05	0.04	0.07	8.49	0.06	0.06	7.95	0.07	0.07
	6.60	0.04	0.07	2.94	0.06	0.07	2.64	0.04	0.04
	7.19	0.04	0.07	4.20	0.06	0.07			
	7.66	0.04	0.07	4.82	0.06	0.07			
	8.18	0.04	0.07	5.43	0.06	0.07			
	3	0.05	0.00	6.03	0.06	0.07			
	6	0.05	0.00	6.69	0.06	0.07			
	9	0.05	0.00	7.29	0.06	0.07			
	3	0.05	0.01	8.04	0.06	0.07			
	6	0.05	0.01	2.86	0.07	0.03			
	9	0.05	0.01	3.55	0.07	0.03			
	3	0.05	0.01	4.10	0.07	0.03			
	6	0.05	0.01	4.64	0.07	0.03			
	9	0.05	0.01	5.20	0.07	0.03			
	6	0.05	0.02	5.88	0.07	0.03			
	9	0.05	0.02	6.41	0.07	0.03			
	3	0.05	0.02	7.04	0.07	0.03			
	6	0.05	0.02	7.59	0.07	0.03			
	9	0.05	0.02	2.87	0.07	0.04			
	3	0.05	0.03	3.44	0.07	0.04			
	6	0.05	0.03	4.06	0.07	0.04			
	9	0.05	0.03	4.69	0.07	0.04			
	3	0.05	0.04	5.29	0.07	0.04			
	6	0.05	0.04	5.83	0.07	0.04			

APPENDIX B: Experimental results (all runs)

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
1.000	0.03	0.03	0.0024	0.0013	0.0011	0.0420	0.0463	0.0436
1.000	0.03	0.03	0.0036	0.0020	0.0016	0.0462	0.0515	0.0476
1.000	0.03	0.03	0.0039	0.0022	0.0017	0.0474	0.0528	0.0484
1.000	0.03	0.03	0.0043	0.0024	0.0019	0.0486	0.0543	0.0497
1.000	0.03	0.03	0.0051	0.0029	0.0022	0.0511	0.0572	0.0517
1.000	0.03	0.03	0.0056	0.0032	0.0024	0.0521	0.0590	0.0528
1.000	0.03	0.03	0.0060	0.0035	0.0025	0.0536	0.0605	0.0539
1.000	0.03	0.03	0.0067	0.0039	0.0028	0.0549	0.0630	0.0554
1.000	0.03	0.03	0.0072	0.0043	0.0030	0.0562	0.0649	0.0567
1.000	0.03	0.03	0.0078	0.0047	0.0032	0.0578	0.0669	0.0576
1.000	0.03	0.03	0.0082	0.0049	0.0033	0.0585	0.0680	0.0586
1.000	0.03	0.03	0.0089	0.0054	0.0035	0.0600	0.0701	0.0598
1.000	0.03	0.07	0.0078	0.0075	0.0003	0.0757	0.0778	0.0775
1.000	0.03	0.07	0.0083	0.0079	0.0005	0.0777	0.0786	0.0795
1.000	0.03	0.07	0.0089	0.0083	0.0006	0.0781	0.0810	0.0808
1.000	0.04	0.04	0.0019	0.0010	0.0009	0.0499	0.0534	0.0514
1.000	0.04	0.04	0.0026	0.0014	0.0012	0.0527	0.0566	0.0544
1.000	0.04	0.04	0.0032	0.0017	0.0015	0.0550	0.0593	0.0564
1.000	0.04	0.04	0.0037	0.0020	0.0017	0.0569	0.0613	0.0581
1.000	0.04	0.04	0.0042	0.0023	0.0019	0.0587	0.0633	0.0600
1.000	0.04	0.04	0.0048	0.0026	0.0022	0.0606	0.0656	0.0613
1.000	0.04	0.04	0.0052	0.0029	0.0023	0.0619	0.0673	0.0627
1.000	0.04	0.04	0.0059	0.0033	0.0026	0.0638	0.0696	0.0645
1.000	0.04	0.04	0.0064	0.0036	0.0028	0.0650	0.0715	0.0652
1.000	0.04	0.04	0.0070	0.0040	0.0030	0.0666	0.0734	0.0669
1.000	0.04	0.04	0.0078	0.0044	0.0034	0.0687	0.0761	0.0687
1.000	0.04	0.04	0.0085	0.0049	0.0037	0.0705	0.0785	0.0710
1.000	0.04	0.04	0.0093	0.0054	0.0039	0.0717	0.0807	0.0719
1.000	0.04	0.07	0.0051	0.0047	0.0004	0.0767	0.0795	0.0791
1.000	0.04	0.07	0.0058	0.0051	0.0006	0.0790	0.0821	0.0812
1.000	0.04	0.07	0.0063	0.0055	0.0008	0.0807	0.0835	0.0831
1.000	0.04	0.07	0.0068	0.0059	0.0009	0.0823	0.0858	0.0853
1.000	0.04	0.07	0.0074	0.0063	0.0011	0.0842	0.0872	0.0865
1.000	0.04	0.07	0.0080	0.0067	0.0013	0.0862	0.0895	0.0880
1.000	0.05	0.00	0.0030	0.0000	0.0030	0.0359	0.0420	0.0286
1.000	0.05	0.00	0.0060	0.0005	0.0055	0.0529	0.0585	0.0397
1.000	0.05	0.01	0.0030	0.0000	0.0030	0.0357	0.0423	0.0289
1.000	0.05	0.01	0.0060	0.0006	0.0054	0.0527	0.0597	0.0400
1.000	0.05	0.01	0.0030	0.0000	0.0030	0.0389	0.0443	0.0348
1.000	0.05	0.01	0.0060	0.0007	0.0053	0.0542	0.0613	0.0433
1.000	0.05	0.02	0.0030	0.0000	0.0030	0.0414	0.0473	0.0389

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
1.000	0.05	0.02	0.0060	0.0007	0.0053	0.0550	0.0624	0.0467
1.000	0.05	0.02	0.0030	0.0000	0.0030	0.0464	0.0518	0.0452
1.000	0.05	0.02	0.0060	0.0011	0.0049	0.0576	0.0647	0.0527
1.000	0.05	0.02	0.0090	0.0031	0.0059	0.0662	0.0771	0.0593
1.000	0.05	0.03	0.0030	0.0004	0.0026	0.0535	0.0588	0.0547
1.000	0.05	0.03	0.0060	0.0018	0.0042	0.0648	0.0711	0.0634
1.000	0.05	0.03	0.0090	0.0034	0.0056	0.0716	0.0812	0.0695
1.000	0.05	0.04	0.0030	0.0011	0.0019	0.0601	0.0646	0.0626
1.000	0.05	0.04	0.0060	0.0026	0.0034	0.0715	0.0774	0.0720
1.000	0.05	0.04	0.0090	0.0042	0.0048	0.0789	0.0866	0.0782
1.000	0.05	0.05	0.0028	0.0016	0.0012	0.0631	0.0667	0.0652
1.000	0.05	0.05	0.0036	0.0020	0.0016	0.0659	0.0701	0.0679
1.000	0.05	0.05	0.0042	0.0024	0.0019	0.0681	0.0727	0.0703
1.000	0.05	0.05	0.0050	0.0028	0.0022	0.0703	0.0756	0.0724
1.000	0.05	0.05	0.0055	0.0031	0.0024	0.0721	0.0774	0.0739
1.000	0.05	0.05	0.0062	0.0035	0.0027	0.0739	0.0797	0.0758
1.000	0.05	0.05	0.0069	0.0039	0.0030	0.0760	0.0824	0.0773
1.000	0.05	0.05	0.0075	0.0043	0.0032	0.0777	0.0844	0.0790
1.000	0.05	0.05	0.0080	0.0046	0.0034	0.0791	0.0859	0.0798
1.000	0.05	0.05	0.0087	0.0051	0.0037	0.0805	0.0888	0.0813
1.000	0.05	0.07	0.0039	0.0032	0.0007	0.0792	0.0830	0.0820
1.000	0.05	0.07	0.0046	0.0037	0.0009	0.0816	0.0858	0.0841
1.000	0.05	0.07	0.0050	0.0040	0.0010	0.0832	0.0876	0.0857
1.000	0.05	0.07	0.0057	0.0044	0.0012	0.0853	0.0893	0.0875
1.000	0.05	0.07	0.0063	0.0048	0.0015	0.0873	0.0908	0.0892
1.000	0.05	0.07	0.0068	0.0052	0.0016	0.0886	0.0929	0.0912
1.000	0.05	0.07	0.0074	0.0056	0.0019	0.0904	0.0951	0.0923
1.000	0.05	0.07	0.0080	0.0060	0.0021	0.0926	0.0969	0.0939
1.000	0.05	0.07	0.0087	0.0063	0.0023	0.0942	0.0986	0.0956
1.000	0.06	0.06	0.0024	0.0013	0.0011	0.0723	0.0753	0.0737
1.000	0.06	0.06	0.0033	0.0018	0.0015	0.0744	0.0790	0.0770
1.000	0.06	0.06	0.0041	0.0022	0.0019	0.0796	0.0830	0.0807
1.000	0.06	0.06	0.0047	0.0025	0.0022	0.0817	0.0855	0.0829
1.000	0.06	0.06	0.0053	0.0029	0.0025	0.0838	0.0875	0.0846
1.000	0.06	0.06	0.0060	0.0032	0.0028	0.0858	0.0900	0.0867
1.000	0.06	0.06	0.0065	0.0035	0.0030	0.0875	0.0915	0.0879
1.000	0.06	0.06	0.0070	0.0038	0.0032	0.0888	0.0933	0.0896
1.000	0.06	0.06	0.0075	0.0041	0.0034	0.0901	0.0950	0.0906
1.000	0.06	0.06	0.0080	0.0044	0.0036	0.0917	0.0964	0.0920
1.000	0.06	0.06	0.0085	0.0046	0.0038	0.0929	0.0981	0.0931
1.000	0.06	0.07	0.0032	0.0024	0.0008	0.0807	0.0845	0.0825

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
1.000	0.06	0.07	0.0038	0.0027	0.0011	0.0832	0.0871	0.0856
1.000	0.06	0.07	0.0044	0.0031	0.0013	0.0854	0.0894	0.0879
1.000	0.06	0.07	0.0051	0.0035	0.0015	0.0878	0.0925	0.0903
1.000	0.06	0.07	0.0057	0.0039	0.0018	0.0896	0.0944	0.0923
1.000	0.06	0.07	0.0063	0.0042	0.0020	0.0915	0.0967	0.0941
1.000	0.06	0.07	0.0069	0.0046	0.0022	0.0934	0.0984	0.0946
1.000	0.06	0.07	0.0075	0.0050	0.0025	0.0953	0.1001	0.0965
1.000	0.06	0.07	0.0080	0.0053	0.0027	0.0966	0.1017	0.0987
1.000	0.07	0.03	0.0027	0.0000	0.0027	0.0569	0.0614	0.0582
1.000	0.07	0.03	0.0033	0.0000	0.0033	0.0607	0.0649	0.0607
1.000	0.07	0.03	0.0038	0.0001	0.0038	0.0644	0.0684	0.0644
1.000	0.07	0.03	0.0045	0.0002	0.0043	0.0684	0.0731	0.0665
1.000	0.07	0.03	0.0050	0.0004	0.0045	0.0699	0.0747	0.0678
1.000	0.07	0.03	0.0056	0.0007	0.0049	0.0720	0.0772	0.0695
1.000	0.07	0.04	0.0030	0.0000	0.0030	0.0687	0.0731	0.0694
1.000	0.07	0.04	0.0038	0.0003	0.0035	0.0714	0.0758	0.0726
1.000	0.07	0.04	0.0043	0.0006	0.0038	0.0737	0.0782	0.0736
1.000	0.07	0.04	0.0049	0.0008	0.0041	0.0758	0.0800	0.0752
1.000	0.07	0.04	0.0055	0.0011	0.0044	0.0777	0.0824	0.0777
1.000	0.07	0.04	0.0061	0.0014	0.0046	0.0794	0.0845	0.0788
1.000	0.07	0.04	0.0066	0.0017	0.0050	0.0813	0.0867	0.0803
1.000	0.07	0.04	0.0072	0.0020	0.0052	0.0800	0.0884	0.0823
1.000	0.07	0.04	0.0078	0.0023	0.0055	0.0855	0.0907	0.0840
1.000	0.07	0.05	0.0030	0.0005	0.0024	0.0741	0.0779	0.0756
1.000	0.07	0.05	0.0036	0.0008	0.0028	0.0764	0.0801	0.0773
1.000	0.07	0.05	0.0041	0.0010	0.0031	0.0786	0.0831	0.0795
1.000	0.07	0.05	0.0048	0.0013	0.0034	0.0808	0.0850	0.0821
1.000	0.07	0.05	0.0053	0.0016	0.0037	0.0826	0.0874	0.0840
1.000	0.07	0.05	0.0060	0.0020	0.0040	0.0846	0.0895	0.0858
1.000	0.07	0.05	0.0065	0.0023	0.0042	0.0860	0.0913	0.0874
1.000	0.07	0.05	0.0071	0.0026	0.0045	0.0877	0.0929	0.0874
1.000	0.07	0.05	0.0076	0.0029	0.0047	0.0891	0.0949	0.0890
1.000	0.07	0.06	0.0030	0.0010	0.0020	0.0062	0.0833	0.0816
1.000	0.07	0.06	0.0036	0.0014	0.0022	0.0061	0.0857	0.0839
1.000	0.07	0.06	0.0042	0.0017	0.0025	0.0060	0.0880	0.0858
1.000	0.07	0.06	0.0049	0.0021	0.0028	0.0059	0.0903	0.0878
1.000	0.07	0.06	0.0056	0.0025	0.0031	0.0058	0.0923	0.0896
1.000	0.07	0.06	0.0061	0.0028	0.0033	0.0057	0.0937	0.0901
1.000	0.07	0.06	0.0068	0.0032	0.0036	0.0057	0.0963	0.0921
1.000	0.07	0.06	0.0073	0.0035	0.0038	0.0056	0.0981	0.0935
1.000	0.07	0.06	0.0079	0.0035	0.0044	0.0054	0.1018	0.0971

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
1.000	0.07	0.07	0.0023	0.0013	0.0011	0.0817	0.0842	0.0829
1.000	0.07	0.07	0.0031	0.0017	0.0014	0.0853	0.0880	0.0863
1.000	0.07	0.07	0.0039	0.0021	0.0018	0.0882	0.0912	0.0892
1.000	0.07	0.07	0.0048	0.0026	0.0022	0.0912	0.0943	0.0920
1.000	0.07	0.07	0.0056	0.0031	0.0025	0.0938	0.0974	0.0944
1.000	0.07	0.07	0.0062	0.0034	0.0028	0.0958	0.0993	0.0962
1.000	0.07	0.07	0.0068	0.0037	0.0031	0.0978	0.1015	0.0982
1.000	0.07	0.07	0.0075	0.0041	0.0034	0.0998	0.1038	0.0998
1.000	0.07	0.07	0.0081	0.0044	0.0036	0.1015	0.1056	0.1014
1.000	0.07	0.07	0.0090	0.0050	0.0040	0.1038	0.1084	0.1036
0.867	0.03	0.03	0.00264	0.00157	0.00107	0.04384	0.04727	0.04381
0.867	0.03	0.03	0.00364	0.00219	0.00145	0.04754	0.05175	0.04718
0.867	0.03	0.03	0.00426	0.00258	0.00168	0.04942	0.05441	0.04898
0.867	0.03	0.03	0.00485	0.00297	0.00188	0.05124	0.05663	0.05075
0.867	0.03	0.03	0.00547	0.00337	0.0021	0.05306	0.0589	0.05238
0.867	0.03	0.03	0.00595	0.0037	0.00225	0.05439	0.06074	0.05371
0.867	0.03	0.03	0.00661	0.00415	0.00246	0.05616	0.06339	0.05478
0.867	0.03	0.03	0.00702	0.00442	0.0026	0.05699	0.06479	0.05534
0.867	0.03	0.03	0.00754	0.00478	0.00276	0.05859	0.06618	0.05639
0.867	0.03	0.03	0.00792	0.00503	0.00289	0.05941	0.06764	0.05808
0.867	0.03	0.07	0.00668	0.00652	0.00016	0.11496	0.07497	0.07461
0.867	0.03	0.07	0.00717	0.00686	0.00031	0.11916	0.07655	0.07578
0.867	0.03	0.07	0.00769	0.00727	0.00042	0.12076	0.07766	0.07723
0.867	0.03	0.07	0.00815	0.00763	0.00052	0.12231	0.07971	0.07829
0.867	0.03	0.07	0.00846	0.00788	0.00058	0.12374	0.08054	0.07937
0.867	0.04	0.04	0.00263	0.00151	0.00112	0.05351	0.05706	0.05363
0.867	0.04	0.04	0.00313	0.0018	0.00133	0.05566	0.05931	0.05559
0.867	0.04	0.04	0.00381	0.00221	0.0016	0.05803	0.0622	0.05814
0.867	0.04	0.04	0.00453	0.00264	0.00189	0.06052	0.06505	0.06023
0.867	0.04	0.04	0.00505	0.00297	0.00208	0.06223	0.06701	0.06157
0.867	0.04	0.04	0.00573	0.00339	0.00234	0.06433	0.06965	0.06356
0.867	0.04	0.04	0.00625	0.00372	0.00253	0.0656	0.07151	0.06469
0.867	0.04	0.04	0.00683	0.00408	0.00275	0.06731	0.07348	0.06675
0.867	0.04	0.04	0.00743	0.00446	0.00297	0.06886	0.07553	0.06746
0.867	0.04	0.04	0.00797	0.0048	0.00317	0.07013	0.07725	0.06895
0.867	0.04	0.07	0.00527	0.00494	0.00033	0.0746	0.07701	0.07612
0.867	0.04	0.07	0.00569	0.00525	0.00044	0.07581	0.07842	0.07731
0.867	0.04	0.07	0.00601	0.00548	0.00054	0.07692	0.07938	0.07814
0.867	0.04	0.07	0.0063	0.0057	0.0006	0.0778	0.08028	0.07917
0.867	0.04	0.07	0.00671	0.00603	0.00069	0.0794	0.08183	0.08073
0.867	0.04	0.07	0.00696	0.00621	0.00076	0.08012	0.08289	0.08138

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.867	0.04	0.07	0.00724	0.00643	0.00081	0.08089	0.08405	0.08223
0.867	0.04	0.07	0.00759	0.00668	0.00091	0.08183	0.0851	0.08328
0.867	0.04	0.07	0.00813	0.00702	0.00111	0.08421	0.08535	0.08733
0.867	0.04	0.07	0.00845	0.00726	0.00119	0.08498	0.08828	0.08618
0.867	0.05	0.00	0.003	2.2E-06	0.003	0.04022	0.04472	0.03164
0.867	0.05	0.00	0.006	0.001	0.005	0.05547	0.06165	0.04236
0.867	0.05	0.01	0.003	3.4E-05	0.00297	0.03943	0.04472	0.03164
0.867	0.05	0.01	0.006	0.00101	0.00499	0.05563	0.06216	0.04279
0.867	0.05	0.01	0.003	3.3E-05	0.00297	0.04196	0.04663	0.03563
0.867	0.05	0.01	0.006	0.00115	0.00485	0.05571	0.06216	0.04425
0.867	0.05	0.02	0.003	4.2E-05	0.00296	0.04524	0.04958	0.04115
0.867	0.05	0.02	0.006	0.00125	0.00475	0.05743	0.06377	0.04945
0.867	0.05	0.02	0.003	0.00012	0.00288	0.05086	0.05509	0.04737
0.867	0.05	0.02	0.006	0.00148	0.00452	0.05975	0.06593	0.05385
0.867	0.05	0.02	0.009	0.00326	0.00574	0.07002	0.07993	0.062
0.867	0.05	0.03	0.003	0.00062	0.00238	0.05522	0.05914	0.05521
0.867	0.05	0.03	0.006	0.00217	0.00383	0.06474	0.07132	0.06269
0.867	0.05	0.03	0.009	0.00383	0.00517	0.0751	0.08446	0.07114
0.867	0.05	0.04	0.003	0.00122	0.00178	0.06042	0.06385	0.06065
0.867	0.05	0.04	0.006	0.00304	0.00296	0.07005	0.07569	0.07089
0.867	0.05	0.04	0.009	0.00455	0.00445	0.08062	0.08766	0.07909
0.867	0.05	0.05	0.00236	0.00152	0.00085	0.06179	0.06514	0.06355
0.867	0.05	0.05	0.00288	0.00185	0.00103	0.06405	0.06755	0.06541
0.867	0.05	0.05	0.00336	0.00216	0.0012	0.06571	0.06943	0.06722
0.867	0.05	0.05	0.00398	0.00256	0.00142	0.06803	0.07171	0.06904
0.867	0.05	0.05	0.00454	0.00294	0.0016	0.06963	0.07371	0.07067
0.867	0.05	0.05	0.00548	0.00358	0.0019	0.07223	0.07682	0.07345
0.867	0.05	0.05	0.00604	0.00397	0.00207	0.07399	0.07874	0.07481
0.867	0.05	0.05	0.00649	0.00393	0.00256	0.07797	0.08308	0.07849
0.867	0.05	0.05	0.00699	0.00425	0.00274	0.07918	0.08484	0.08002
0.867	0.05	0.05	0.00747	0.00456	0.00291	0.08156	0.08658	0.08145
0.867	0.05	0.07	0.0041	0.00351	0.00059	0.07775	0.08068	0.07913
0.867	0.05	0.07	0.00467	0.0039	0.00077	0.07979	0.08296	0.08131
0.867	0.05	0.07	0.00509	0.00419	0.0009	0.08134	0.08458	0.08275
0.867	0.05	0.07	0.00553	0.00449	0.00104	0.0831	0.08616	0.08427
0.867	0.05	0.07	0.00607	0.00485	0.00122	0.08504	0.08792	0.08602
0.867	0.05	0.07	0.00659	0.00522	0.00137	0.08658	0.08996	0.08756
0.867	0.05	0.07	0.00705	0.00553	0.00152	0.08791	0.09162	0.08885
0.867	0.05	0.07	0.00745	0.0058	0.00165	0.08912	0.09272	0.09023
0.867	0.06	0.06	0.0028	0.0017	0.0011	0.07432	0.07692	0.07482
0.867	0.06	0.06	0.00338	0.00203	0.00135	0.07631	0.0796	0.07703

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.867	0.06	0.06	0.00377	0.00226	0.00151	0.0799	0.08125	0.07859
0.867	0.06	0.06	0.00438	0.00262	0.00176	0.08056	0.08379	0.08062
0.867	0.06	0.06	0.00491	0.00295	0.00196	0.0825	0.0857	0.0825
0.867	0.06	0.06	0.00549	0.0033	0.00219	0.08454	0.08795	0.08433
0.867	0.06	0.06	0.006	0.0036	0.0024	0.08625	0.08978	0.086
0.867	0.06	0.06	0.00659	0.00396	0.00263	0.08791	0.09166	0.08727
0.867	0.06	0.06	0.00712	0.00429	0.00283	0.0894	0.09365	0.08891
0.867	0.06	0.06	0.00754	0.00453	0.00301	0.09067	0.09506	0.09064
0.867	0.06	0.07	0.0045	0.00322	0.00128	0.08653	0.08941	0.08747
0.867	0.06	0.07	0.00492	0.00348	0.00144	0.0873	0.09119	0.0902
0.867	0.06	0.07	0.00533	0.00375	0.00158	0.08857	0.09253	0.09162
0.867	0.06	0.07	0.006	0.00417	0.00183	0.09078	0.09522	0.09337
0.867	0.06	0.07	0.00652	0.00446	0.00206	0.09255	0.09691	0.0954
0.867	0.06	0.07	0.007	0.0048	0.0022	0.09404	0.09856	0.09648
0.867	0.06	0.07	0.00772	0.00523	0.00249	0.09608	0.10116	0.09883
0.867	0.06	0.07	0.00811	0.00548	0.00263	0.09724	0.10219	0.10008
0.867	0.06	0.07	0.0084	0.00569	0.00271	0.09835	0.10296	0.10075
0.867	0.06	0.07	0.00891	0.006	0.00291	0.09945	0.10466	0.10227
0.867	0.07	0.03	0.00313	0	0.00313	0.00708	0.06606	0.06112
0.867	0.07	0.03	0.00376	5.7E-05	0.0037	0.00684	0.07045	0.0646
0.867	0.07	0.03	0.00441	0.00012	0.00429	0.00664	0.07416	0.06719
0.867	0.07	0.03	0.005	0.00041	0.0046	0.00656	0.07759	0.06897
0.867	0.07	0.03	0.00558	0.00063	0.00495	0.00645	0.0797	0.07056
0.867	0.07	0.03	0.00616	0.00086	0.0053	0.00637	0.08191	0.07196
0.867	0.07	0.03	0.00684	0.00122	0.00562	0.00624	0.08398	0.0745
0.867	0.07	0.03	0.00732	0.00171	0.0056	0.00616	0.08641	0.07597
0.867	0.07	0.03	0.00793	0.0019	0.00603	0.00605	0.08931	0.07875
0.867	0.07	0.04	0.00292	0	0.00292	0.0067	0.07283	0.0696
0.867	0.07	0.04	0.00409	0.00051	0.00358	0.00646	0.07856	0.07406
0.867	0.07	0.04	0.00463	0.00075	0.00388	0.00638	0.08046	0.0753
0.867	0.07	0.04	0.00528	0.00107	0.00421	0.00626	0.08318	0.07687
0.867	0.07	0.04	0.00583	0.00139	0.00444	0.00617	0.08559	0.07865
0.867	0.07	0.04	0.00647	0.00171	0.00476	0.00608	0.08799	0.08105
0.867	0.07	0.04	0.00704	0.00204	0.00499	0.00599	0.0899	0.0826
0.867	0.07	0.04	0.00769	0.0024	0.00529	0.00586	0.09208	0.08439
0.867	0.07	0.05	0.0033	0.00073	0.00256	0.00633	0.08146	0.07834
0.867	0.07	0.05	0.00397	0.00115	0.00282	0.00621	0.08385	0.08034
0.867	0.07	0.05	0.00458	0.00144	0.00313	0.0061	0.08576	0.08276
0.867	0.07	0.05	0.00524	0.00183	0.00341	0.006	0.08859	0.08435
0.867	0.07	0.05	0.00585	0.00217	0.00368	0.00591	0.09085	0.08667
0.867	0.07	0.05	0.00646	0.00251	0.00394	0.00582	0.09301	0.08829

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.867	0.07	0.05	0.00704	0.00287	0.00417	0.00573	0.09547	0.08991
0.867	0.07	0.05	0.00755	0.00319	0.00436	0.00567	0.09743	0.09061
0.867	0.07	0.05	0.00798	0.00343	0.00455	0.00557	0.09854	0.09124
0.867	0.07	0.07	0.00249	0.00159	0.0009	0.08244	0.08486	0.08284
0.867	0.07	0.07	0.00313	0.00199	0.00114	0.08531	0.08787	0.08564
0.867	0.07	0.07	0.00368	0.00229	0.00139	0.08802	0.09028	0.08772
0.867	0.07	0.07	0.00435	0.00271	0.00164	0.09028	0.09277	0.09053
0.867	0.07	0.07	0.0049	0.00305	0.00185	0.09232	0.09507	0.09225
0.867	0.07	0.07	0.00539	0.00334	0.00205	0.09398	0.09692	0.09359
0.867	0.07	0.07	0.00588	0.00362	0.00226	0.09547	0.09879	0.09562
0.867	0.07	0.07	0.00641	0.00395	0.00246	0.09729	0.10057	0.09654
0.867	0.07	0.07	0.00689	0.00424	0.00265	0.09884	0.1022	0.09845
0.867	0.07	0.07	0.00758	0.00465	0.00293	0.1011	0.10445	0.10037
0.733	0.03	0.03	0.00277	0.00179	0.00098	0.0471	0.05258	0.04945
0.733	0.03	0.03	0.00345	0.00222	0.00123	0.04985	0.05545	0.05161
0.733	0.03	0.03	0.00407	0.00265	0.00141	0.05226	0.05785	0.05411
0.733	0.03	0.03	0.00459	0.00301	0.00158	0.05393	0.0606	0.0557
0.733	0.03	0.03	0.00525	0.00346	0.00179	0.05613	0.0628	0.05745
0.733	0.03	0.03	0.00579	0.00385	0.00194	0.05756	0.06497	0.0596
0.733	0.03	0.03	0.00646	0.00431	0.00215	0.05967	0.06687	0.06108
0.733	0.03	0.03	0.00704	0.0047	0.00234	0.06126	0.06899	0.06207
0.733	0.03	0.03	0.00764	0.00511	0.00253	0.06335	0.0709	0.06382
0.733	0.03	0.07	0.00776	0.00752	0.00024	0.05185	0.07794	0.07733
0.733	0.03	0.07	0.00821	0.00786	0.00035	0.0767	0.07915	0.07855
0.733	0.03	0.07	0.00871	0.00825	0.00046	0.07797	0.08001	0.07979
0.733	0.03	0.07	0.0091	0.0086	0.0005	0.07858	0.08146	0.08106
0.733	0.03	0.07	0.00964	0.00905	0.00059	0.08073	0.08346	0.08248
0.733	0.04	0.04	0.00234	0.00151	0.00083	0.05295	0.05595	0.0539
0.733	0.04	0.04	0.00297	0.00192	0.00105	0.05566	0.05922	0.05667
0.733	0.04	0.04	0.00367	0.00236	0.00131	0.05831	0.06234	0.0592
0.733	0.04	0.04	0.00408	0.00264	0.00144	0.05991	0.06409	0.06078
0.733	0.04	0.04	0.00445	0.00288	0.00157	0.06135	0.06588	0.06174
0.733	0.04	0.04	0.00503	0.00342	0.00161	0.06218	0.06663	0.06269
0.733	0.04	0.04	0.00561	0.00381	0.0018	0.06416	0.06911	0.06433
0.733	0.04	0.04	0.00608	0.00414	0.00194	0.0656	0.07091	0.0658
0.733	0.04	0.04	0.00672	0.00458	0.00214	0.06737	0.07302	0.0673
0.733	0.04	0.04	0.00738	0.00503	0.00235	0.06957	0.07559	0.06911
0.733	0.04	0.07	0.00567	0.00547	0.0002	0.07521	0.07767	0.07669
0.733	0.04	0.07	0.0063	0.00591	0.00039	0.07731	0.07979	0.07904
0.733	0.04	0.07	0.00705	0.00651	0.00054	0.07973	0.08247	0.08156
0.733	0.04	0.07	0.00764	0.00698	0.00066	0.08161	0.08441	0.0826

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.733	0.04	0.07	0.00824	0.00744	0.0008	0.08382	0.08677	0.08479
0.733	0.04	0.07	0.009	0.00803	0.00098	0.08531	0.08849	0.08712
0.733	0.04	0.07	0.00959	0.00848	0.00111	0.08769	0.09125	0.08883
0.733	0.05	0.00	0.003	3E-05	0.00297	0.04499	0.05068	0.03509
0.733	0.05	0.00	0.006	0.00154	0.00446	0.05848	0.06488	0.04553
0.733	0.05	0.00	0.009	0.00315	0.00585	0.07097	0.0793	0.0538
0.733	0.05	0.01	0.003	3.1E-05	0.00297	0.04497	0.0507	0.03583
0.733	0.05	0.01	0.006	0.00143	0.00457	0.05975	0.06587	0.04724
0.733	0.05	0.01	0.009	0.00317	0.00583	0.07171	0.0794	0.0538
0.733	0.05	0.01	0.003	1.2E-05	0.00299	0.04562	0.05145	0.03702
0.733	0.05	0.01	0.006	0.00156	0.00444	0.06035	0.06712	0.04968
0.733	0.05	0.01	0.009	0.00319	0.00581	0.07178	0.08039	0.05676
0.733	0.05	0.02	0.003	0.00014	0.00286	0.0506	0.05522	0.0462
0.733	0.05	0.02	0.009	0.00352	0.00548	0.07369	0.08167	0.06156
0.733	0.05	0.02	0.003	0.00039	0.00261	0.05306	0.05744	0.05065
0.733	0.05	0.02	0.006	0.00201	0.00399	0.06488	0.07108	0.0596
0.733	0.05	0.02	0.009	0.00377	0.00523	0.07679	0.0829	0.06717
0.733	0.05	0.03	0.003	0.00038	0.00262	0.0566	0.06161	0.05644
0.733	0.05	0.03	0.006	0.00256	0.00344	0.06957	0.07518	0.06716
0.733	0.05	0.03	0.009	0.00444	0.00456	0.08075	0.08745	0.07498
0.733	0.05	0.04	0.003	0.00142	0.00158	0.06228	0.06702	0.06345
0.733	0.05	0.04	0.006	0.00314	0.00286	0.07399	0.07895	0.07309
0.733	0.05	0.04	0.009	0.00497	0.00403	0.08214	0.09063	0.08138
0.733	0.05	0.05	0.00252	0.00152	0.001	0.06162	0.06498	0.06277
0.733	0.05	0.05	0.00331	0.00201	0.0013	0.06516	0.06875	0.06616
0.733	0.05	0.05	0.00412	0.00253	0.00159	0.06842	0.07227	0.0691
0.733	0.05	0.05	0.00471	0.00291	0.0018	0.07068	0.07466	0.07116
0.733	0.05	0.05	0.00541	0.00335	0.00206	0.07289	0.07737	0.07348
0.733	0.05	0.05	0.00584	0.00392	0.00192	0.0714	0.07629	0.07216
0.733	0.05	0.05	0.00634	0.00395	0.00239	0.07581	0.081	0.07607
0.733	0.05	0.05	0.00716	0.00468	0.00248	0.07813	0.08385	0.07876
0.733	0.05	0.05	0.00786	0.005	0.00286	0.1	0.08727	0.07993
0.733	0.05	0.05	0.00848	0.00544	0.00304	0.10436	0.08934	0.08162
0.733	0.05	0.07	0.00367	0.00342	0.00025	0.07543	0.07831	0.07725
0.733	0.05	0.07	0.00423	0.00382	0.00041	0.07775	0.0803	0.0794
0.733	0.05	0.07	0.00479	0.00424	0.00055	0.07996	0.08308	0.08158
0.733	0.05	0.07	0.00535	0.00466	0.00069	0.08216	0.0852	0.08385
0.733	0.05	0.07	0.00584	0.00501	0.00083	0.08404	0.08703	0.08553
0.733	0.05	0.07	0.00618	0.00526	0.00092	0.0852	0.0883	0.08657
0.733	0.05	0.07	0.00653	0.00552	0.00101	0.08642	0.08944	0.08765
0.733	0.06	0.06	0.00266	0.00171	0.00095	0.07261	0.07645	0.07451

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.733	0.06	0.06	0.00304	0.00178	0.00126	0.07609	0.08026	0.07796
0.733	0.06	0.06	0.00438	0.00285	0.00153	0.07902	0.08319	0.08074
0.733	0.06	0.06	0.00513	0.00336	0.00177	0.08161	0.0859	0.08273
0.733	0.06	0.06	0.00583	0.00383	0.002	0.08377	0.08859	0.08515
0.733	0.06	0.06	0.00655	0.00402	0.00253	0.08916	0.09387	0.0894
0.733	0.06	0.06	0.0072	0.00443	0.00277	0.09128	0.09616	0.09156
0.733	0.06	0.06	0.00788	0.00486	0.00302	0.0931	0.09847	0.09333
0.733	0.06	0.06	0.00845	0.00522	0.00323	0.09481	0.09525	0.10048
0.733	0.06	0.07	0.00239	0.00209	0.0003	0.07604	0.07908	0.07791
0.733	0.06	0.07	0.00307	0.00254	0.00053	0.07924	0.08227	0.08103
0.733	0.06	0.07	0.00385	0.00311	0.00074	0.08294	0.08601	0.08445
0.733	0.06	0.07	0.0045	0.00355	0.00095	0.08548	0.0886	0.08685
0.733	0.06	0.07	0.0052	0.00403	0.00117	0.08802	0.09142	0.08928
0.733	0.06	0.07	0.00594	0.00453	0.00141	0.08835	0.09413	0.0919
0.733	0.06	0.07	0.00655	0.00494	0.00161	0.09282	0.09652	0.09373
0.733	0.06	0.07	0.00719	0.00538	0.00181	0.09475	0.0987	0.09588
0.733	0.06	0.07	0.00801	0.00596	0.00205	0.09768	0.10139	0.0985
0.733	0.07	0.03	0.00316	0	0.00316	0.06601	0.07028	0.06429
0.733	0.07	0.03	0.00378	0.00014	0.00364	0.07055	0.0749	0.06792
0.733	0.07	0.03	0.00442	0.00046	0.00395	0.07307	0.0782	0.06972
0.733	0.07	0.03	0.00508	0.00097	0.00411	0.07605	0.08095	0.07285
0.733	0.07	0.03	0.00564	0.00097	0.00467	0.07846	0.08312	0.07519
0.733	0.07	0.03	0.00623	0.00124	0.00499	0.08077	0.0861	0.07652
0.733	0.07	0.03	0.00688	0.00185	0.00503	0.0838	0.08916	0.07753
0.733	0.07	0.03	0.00739	0.00185	0.00554	0.08562	0.09138	0.07952
0.733	0.07	0.03	0.00806	0.00228	0.00578	0.08816	0.09394	0.08125
0.733	0.07	0.04	0.00333	0.00032	0.003	0.07434	0.07927	0.07507
0.733	0.07	0.04	0.00389	0.00073	0.00316	0.07711	0.08112	0.07611
0.733	0.07	0.04	0.00452	0.00101	0.00351	0.07978	0.08368	0.0793
0.733	0.07	0.04	0.00511	0.00132	0.00379	0.08241	0.08686	0.08167
0.733	0.07	0.04	0.0057	0.00162	0.00408	0.08468	0.08956	0.08337
0.733	0.07	0.04	0.00628	0.00194	0.00435	0.087	0.09192	0.08439
0.733	0.07	0.04	0.00687	0.00227	0.0046	0.08911	0.09471	0.08648
0.733	0.07	0.04	0.00735	0.00252	0.00483	0.09084	0.09646	0.0879
0.733	0.07	0.04	0.00796	0.00287	0.00509	0.093	0.09865	0.08934
0.733	0.07	0.05	0.00318	0.00098	0.00219	0.08033	0.08467	0.08144
0.733	0.07	0.05	0.00379	0.00131	0.00248	0.08321	0.0877	0.08392
0.733	0.07	0.05	0.00439	0.00162	0.00277	0.08564	0.09038	0.08658
0.733	0.07	0.05	0.00502	0.00198	0.00305	0.08835	0.09289	0.0889
0.733	0.07	0.05	0.0056	0.00231	0.00329	0.09035	0.0956	0.09056
0.733	0.07	0.05	0.00623	0.00265	0.00358	0.09271	0.09761	0.09269

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.733	0.07	0.05	0.00681	0.00298	0.00382	0.09479	0.09965	0.09435
0.733	0.07	0.05	0.00743	0.00407	0.00336	0.09694	0.10218	0.09588
0.733	0.07	0.05	0.00807	0.00373	0.00434	0.09913	0.10439	0.09807
0.733	0.07	0.06	0.00304	0.00135	0.00169	0.08418	0.08805	0.08621
0.733	0.07	0.06	0.00368	0.00173	0.00196	0.08796	0.09076	0.08805
0.733	0.07	0.06	0.00433	0.00212	0.00221	0.08951	0.09363	0.09135
0.733	0.07	0.06	0.00495	0.00247	0.00248	0.09188	0.09619	0.0932
0.733	0.07	0.06	0.00542	0.00275	0.00267	0.09371	0.09841	0.09521
0.733	0.07	0.06	0.00623	0.00323	0.00301	0.09654	0.10136	0.09724
0.733	0.07	0.06	0.00694	0.00366	0.00328	0.09906	0.10421	0.0999
0.733	0.07	0.06	0.00757	0.00405	0.00352	0.10126	0.10626	0.10102
0.733	0.07	0.06	0.00823	0.00445	0.00377	0.1035	0.10819	0.10322
0.733	0.07	0.07	0.00252	0.00156	0.00096	0.08515	0.08767	0.08613
0.733	0.07	0.07	0.00305	0.00187	0.00118	0.08763	0.09024	0.08839
0.733	0.07	0.07	0.00352	0.00215	0.00137	0.08951	0.0924	0.09063
0.733	0.07	0.07	0.00409	0.00252	0.00157	0.09188	0.09484	0.09285
0.733	0.07	0.07	0.00449	0.00275	0.00174	0.08851	0.09664	0.09439
0.733	0.07	0.07	0.00533	0.00328	0.00205	0.09669	0.09996	0.09733
0.733	0.07	0.07	0.00604	0.00373	0.00231	0.09934	0.10264	0.09969
0.733	0.07	0.07	0.00678	0.00419	0.00259	0.10182	0.10543	0.10187
0.733	0.07	0.07	0.00737	0.00454	0.00283	0.10381	0.1074	0.10394
0.733	0.07	0.07	0.00799	0.00495	0.00304	0.10569	0.10938	0.1057
0.600	0.03	0.03	0.00269	0.00197	0.00072	0.04257	0.04624	0.04346
0.600	0.03	0.03	0.00342	0.00252	0.0009	0.04511	0.04928	0.04599
0.600	0.03	0.03	0.00416	0.00292	0.00124	0.05008	0.05527	0.04978
0.600	0.03	0.03	0.00484	0.00342	0.00142	0.05257	0.05235	0.05786
0.600	0.03	0.03	0.00547	0.00389	0.00158	0.05422	0.06041	0.05396
0.600	0.03	0.03	0.00631	0.00448	0.00183	0.0571	0.06366	0.05623
0.600	0.03	0.03	0.00693	0.00496	0.00197	0.05903	0.06641	0.05774
0.600	0.03	0.03	0.00768	0.00551	0.00217	0.06118	0.06893	0.05871
0.600	0.03	0.03	0.00816	0.00588	0.00228	0.06207	0.07045	0.06051
0.600	0.03	0.03	0.00859	0.00617	0.00242	0.06367	0.07171	0.06098
0.600	0.03	0.07	0.00612	0.00591	0.00021	0.0725	0.075	0.07536
0.600	0.03	0.07	0.00671	0.00637	0.00034	0.07432	0.07761	0.0769
0.600	0.03	0.07	0.00731	0.00687	0.00044	0.07631	0.07939	0.07901
0.600	0.03	0.07	0.00802	0.00746	0.00056	0.07874	0.08148	0.08102
0.600	0.03	0.07	0.00863	0.00798	0.00066	0.07951	0.08394	0.08296
0.600	0.03	0.07	0.00921	0.00844	0.00078	0.08211	0.08636	0.08479
0.600	0.04	0.04	0.0028	0.002	0.0008	0.0545	0.05906	0.05692
0.600	0.04	0.04	0.00348	0.00247	0.00101	0.05726	0.06206	0.06
0.600	0.04	0.04	0.00407	0.00285	0.00122	0.06002	0.06542	0.0618

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.600	0.04	0.04	0.00482	0.00341	0.00141	0.06278	0.06787	0.06467
0.600	0.04	0.04	0.00544	0.00386	0.00158	0.06499	0.07077	0.0665
0.600	0.04	0.04	0.00612	0.00434	0.00178	0.0672	0.07275	0.06799
0.600	0.04	0.04	0.00675	0.0048	0.00195	0.06941	0.07471	0.07009
0.600	0.04	0.04	0.00729	0.00518	0.00211	0.07051	0.07689	0.07186
0.600	0.04	0.04	0.00811	0.00577	0.00234	0.07383	0.07975	0.07323
0.600	0.04	0.07	0.004	0.0037	0.0003	0.07383	0.07802	0.07662
0.600	0.04	0.07	0.00459	0.00415	0.00044	0.07642	0.08015	0.07929
0.600	0.04	0.07	0.00525	0.00467	0.00058	0.07907	0.08298	0.08143
0.600	0.04	0.07	0.00584	0.00513	0.00071	0.08139	0.08522	0.08338
0.600	0.04	0.07	0.00641	0.00557	0.00085	0.08349	0.08664	0.08576
0.600	0.04	0.07	0.00704	0.00606	0.00098	0.08548	0.08893	0.08783
0.600	0.04	0.07	0.00765	0.00652	0.00114	0.08747	0.09107	0.08924
0.600	0.04	0.07	0.00821	0.00694	0.00128	0.08973	0.09313	0.09163
0.600	0.04	0.07	0.00882	0.0074	0.00142	0.09144	0.09519	0.0931
0.600	0.05	0.00	0.003	8.8E-05	0.00291	0.05134	0.05633	0.04058
0.600	0.05	0.00	0.006	0.00203	0.00397	0.0598	0.06615	0.0477
0.600	0.05	0.00	0.009	0.00371	0.00529	0.07581	0.08455	0.05854
0.600	0.05	0.01	0.003	9.4E-05	0.00291	0.05128	0.05633	0.04
0.600	0.05	0.01	0.006	0.00209	0.00391	0.06278	0.0697	0.04898
0.600	0.05	0.01	0.009	0.00372	0.00528	0.07686	0.08487	0.0586
0.600	0.05	0.01	0.003	0.00018	0.00282	0.05174	0.05696	0.04187
0.600	0.05	0.01	0.006	0.00214	0.00386	0.06306	0.06984	0.04941
0.600	0.05	0.01	0.009	0.00379	0.00521	0.07587	0.0852	0.0599
0.600	0.05	0.02	0.003	0.00039	0.00261	0.05407	0.05897	0.04878
0.600	0.05	0.02	0.009	0.00402	0.00498	0.07858	0.08641	0.06539
0.600	0.05	0.02	0.003	0.00058	0.00242	0.05617	0.06106	0.0528
0.600	0.05	0.02	0.006	0.00246	0.00354	0.06593	0.07226	0.05807
0.600	0.05	0.02	0.009	0.00417	0.00483	0.08007	0.08774	0.06959
0.600	0.05	0.03	0.003	0.00099	0.00201	0.05978	0.06483	0.06
0.600	0.05	0.03	0.006	0.00291	0.00309	0.0693	0.07499	0.06514
0.600	0.05	0.03	0.009	0.00474	0.00426	0.08366	0.09094	0.07741
0.600	0.05	0.04	0.003	0.00158	0.00142	0.06436	0.06937	0.06541
0.600	0.05	0.04	0.006	0.00343	0.00257	0.07305	0.07847	0.07139
0.600	0.05	0.04	0.009	0.00536	0.00364	0.08785	0.09425	0.08369
0.600	0.05	0.05	0.00308	0.00209	0.00099	0.06554	0.06915	0.06776
0.600	0.05	0.05	0.00385	0.0026	0.00125	0.06886	0.07277	0.07136
0.600	0.05	0.05	0.00445	0.00301	0.00144	0.07162	0.0765	0.07369
0.600	0.05	0.05	0.00507	0.00343	0.00164	0.07383	0.07883	0.076
0.600	0.05	0.05	0.00577	0.00391	0.00186	0.07604	0.08186	0.07799
0.600	0.05	0.05	0.00647	0.00439	0.00208	0.0788	0.08435	0.08055

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.600	0.05	0.05	0.00717	0.0049	0.00227	0.081	0.08682	0.08243
0.600	0.05	0.05	0.00784	0.00537	0.00247	0.08321	0.08943	0.08446
0.600	0.05	0.05	0.00839	0.00576	0.00263	0.08432	0.0912	0.08563
0.600	0.05	0.07	0.00302	0.00257	0.00045	0.07703	0.08059	0.07964
0.600	0.05	0.07	0.00364	0.00301	0.00063	0.08007	0.08348	0.08264
0.600	0.05	0.07	0.00425	0.00346	0.00079	0.08272	0.08624	0.08523
0.600	0.05	0.07	0.00483	0.00388	0.00095	0.08509	0.08855	0.08733
0.600	0.05	0.07	0.00548	0.00436	0.00112	0.08752	0.09182	0.08954
0.600	0.05	0.07	0.00607	0.00494	0.00113	0.08973	0.09341	0.0918
0.600	0.05	0.07	0.00662	0.00518	0.00144	0.09161	0.09499	0.09336
0.600	0.05	0.07	0.0072	0.00559	0.00161	0.09365	0.09718	0.09535
0.600	0.05	0.07	0.00779	0.00602	0.00177	0.09564	0.09947	0.09722
0.600	0.05	0.07	0.00835	0.00642	0.00193	0.09707	0.10127	0.09876
0.600	0.06	0.06	0.00258	0.0017	0.00088	0.0741	0.07851	0.0772
0.600	0.06	0.06	0.00317	0.00208	0.00109	0.07703	0.08158	0.07976
0.600	0.06	0.06	0.00372	0.00245	0.00127	0.07957	0.08416	0.08192
0.600	0.06	0.06	0.00444	0.00313	0.00131	0.08023	0.08496	0.0825
0.600	0.06	0.06	0.00504	0.00357	0.00147	0.08216	0.086	0.08443
0.600	0.06	0.06	0.0056	0.00373	0.00187	0.08719	0.09132	0.08869
0.600	0.06	0.06	0.00621	0.00413	0.00208	0.0894	0.09418	0.09097
0.600	0.06	0.06	0.00694	0.00464	0.0023	0.09188	0.09665	0.09329
0.600	0.06	0.06	0.00752	0.00502	0.0025	0.09409	0.09882	0.09499
0.600	0.06	0.06	0.00816	0.00548	0.00268	0.09608	0.10086	0.09699
0.600	0.06	0.07	0.00308	0.00225	0.00083	0.08327	0.08619	0.08526
0.600	0.06	0.07	0.00367	0.00266	0.00101	0.08609	0.0896	0.08829
0.600	0.06	0.07	0.00423	0.00303	0.0012	0.08863	0.09258	0.09094
0.600	0.06	0.07	0.00485	0.00347	0.00138	0.09078	0.09468	0.09239
0.600	0.06	0.07	0.00542	0.00386	0.00156	0.0931	0.09771	0.09529
0.600	0.06	0.07	0.00599	0.00425	0.00174	0.09514	0.09955	0.09686
0.600	0.06	0.07	0.00659	0.00467	0.00192	0.09735	0.10164	0.09916
0.600	0.06	0.07	0.00717	0.00506	0.00211	0.09945	0.10412	0.1008
0.600	0.06	0.07	0.00774	0.00545	0.00229	0.10121	0.10623	0.10182
0.600	0.06	0.07	0.00817	0.00576	0.00241	0.10282	0.10767	0.10385
0.600	0.07	0.03	0.00271	0.00001	0.0027	0.06549	0.06978	0.06265
0.600	0.07	0.03	0.00329	0.00017	0.00312	0.07002	0.07448	0.06697
0.600	0.07	0.03	0.00402	0.00046	0.00356	0.07427	0.0784	0.06941
0.600	0.07	0.03	0.00456	0.00058	0.00398	0.07808	0.08223	0.07251
0.600	0.07	0.03	0.0052	0.00093	0.00427	0.08089	0.08478	0.07442
0.600	0.07	0.03	0.00577	0.00124	0.00453	0.08327	0.0876	0.07566
0.600	0.07	0.03	0.00626	0.00155	0.00471	0.08515	0.09029	0.07697
0.600	0.07	0.03	0.00695	0.002	0.00495	0.08785	0.09318	0.07887

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.600	0.07	0.03	0.00757	0.00257	0.005	0.08934	0.09549	0.08021
0.600	0.07	0.04	0.00278	0.00042	0.00236	0.07118	0.07531	0.07091
0.600	0.07	0.04	0.00342	0.00077	0.00265	0.07436	0.07866	0.07381
0.600	0.07	0.04	0.00426	0.00129	0.00297	0.07797	0.08258	0.0767
0.600	0.07	0.04	0.00486	0.00164	0.00322	0.08034	0.08542	0.07847
0.600	0.07	0.04	0.00539	0.00196	0.00343	0.08261	0.08725	0.07992
0.600	0.07	0.04	0.00587	0.00228	0.00359	0.08432	0.08926	0.08148
0.600	0.07	0.04	0.00633	0.00261	0.00372	0.0857	0.09078	0.0827
0.600	0.07	0.04	0.00687	0.00293	0.00394	0.08758	0.09309	0.08383
0.600	0.07	0.04	0.00737	0.00325	0.00412	0.08934	0.09506	0.0855
0.600	0.07	0.04	0.00809	0.0035	0.00459	0.09415	0.09955	0.08932
0.600	0.07	0.05	0.00309	0.00085	0.00224	0.08222	0.08532	0.08299
0.600	0.07	0.05	0.00371	0.0012	0.00251	0.08542	0.08933	0.08541
0.600	0.07	0.05	0.00427	0.00151	0.00276	0.08807	0.0921	0.08792
0.600	0.07	0.05	0.00487	0.00185	0.00302	0.09056	0.09477	0.09022
0.600	0.07	0.05	0.00552	0.00225	0.00327	0.09326	0.09827	0.09209
0.600	0.07	0.05	0.00613	0.00261	0.00352	0.09575	0.10037	0.09423
0.600	0.07	0.05	0.00666	0.00292	0.00374	0.09774	0.10285	0.09556
0.600	0.07	0.05	0.00732	0.00335	0.00397	0.10033	0.105	0.09782
0.600	0.07	0.05	0.00798	0.0037	0.00428	0.10254	0.10794	0.09903
0.600	0.07	0.05	0.00841	0.00401	0.0044	0.10381	0.10956	0.10093
0.600	0.07	0.06	0.00286	0.00116	0.0017	0.08542	0.08902	0.08684
0.600	0.07	0.06	0.00343	0.00148	0.00195	0.08824	0.09244	0.0894
0.600	0.07	0.06	0.00403	0.00185	0.00218	0.09083	0.09443	0.09208
0.600	0.07	0.06	0.00458	0.00217	0.00241	0.09337	0.09785	0.09424
0.600	0.07	0.06	0.00512	0.00252	0.0026	0.09564	0.10001	0.09589
0.600	0.07	0.06	0.00567	0.00286	0.00281	0.0979	0.10237	0.09758
0.600	0.07	0.06	0.00621	0.00317	0.00304	0.1	0.10477	0.09978
0.600	0.07	0.06	0.0068	0.00355	0.00325	0.10226	0.10708	0.1014
0.600	0.07	0.06	0.00737	0.00392	0.00345	0.10431	0.10917	0.10376
0.600	0.07	0.06	0.00807	0.00438	0.00369	0.10674	0.11191	0.10561
0.600	0.07	0.07	0.00269	0.00165	0.00104	0.08487	0.08883	0.08733
0.600	0.07	0.07	0.00328	0.00202	0.00126	0.08774	0.09124	0.0892
0.600	0.07	0.07	0.00392	0.00244	0.00148	0.0905	0.09414	0.09219
0.600	0.07	0.07	0.00453	0.00283	0.0017	0.09321	0.0976	0.09486
0.600	0.07	0.07	0.0051	0.00321	0.00189	0.09547	0.1	0.09701
0.600	0.07	0.07	0.00573	0.00363	0.0021	0.09785	0.10242	0.09919
0.600	0.07	0.07	0.00631	0.004	0.00231	0.10017	0.10498	0.10156
0.600	0.07	0.07	0.00696	0.00444	0.00252	0.10248	0.10718	0.10312
0.600	0.07	0.07	0.00751	0.00482	0.00269	0.10436	0.10826	0.10471
0.600	0.07	0.07	0.00813	0.00524	0.00289	0.10646	0.11114	0.10656

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.333	0.03	0.03	0.00299	0.00237	0.00063	0.04666	0.05094	0.04766
0.333	0.03	0.03	0.00364	0.00288	0.00076	0.04986	0.05444	0.04996
0.333	0.03	0.03	0.00432	0.00342	0.0009	0.05262	0.05728	0.0525
0.333	0.03	0.03	0.00514	0.00406	0.00108	0.05605	0.06092	0.05494
0.333	0.03	0.03	0.0057	0.00453	0.00117	0.0582	0.0634	0.05703
0.333	0.03	0.03	0.0062	0.00493	0.00127	0.06002	0.06522	0.05858
0.333	0.03	0.03	0.00683	0.00545	0.00138	0.06201	0.0677	0.06
0.333	0.03	0.03	0.00766	0.00614	0.00152	0.06477	0.07056	0.0628
0.333	0.03	0.03	0.0084	0.00676	0.00164	0.06681	0.07317	0.06411
0.333	0.03	0.07	0.00586	0.00563	0.00023	0.07151	0.07513	0.0742
0.333	0.03	0.07	0.00649	0.0062	0.00029	0.07421	0.07729	0.07697
0.333	0.03	0.07	0.00709	0.00674	0.00035	0.07626	0.0794	0.07836
0.333	0.03	0.07	0.00765	0.00723	0.00042	0.07863	0.08159	0.07998
0.333	0.03	0.07	0.00822	0.00775	0.00047	0.08062	0.0833	0.08247
0.333	0.03	0.07	0.00878	0.00823	0.00056	0.08432	0.08518	0.08427
0.333	0.03	0.07	0.00941	0.00877	0.00064	0.08531	0.08812	0.08616
0.333	0.04	0.04	0.0034	0.00261	0.00079	0.05941	0.06355	0.05993
0.333	0.04	0.04	0.00399	0.00306	0.00093	0.0624	0.06633	0.06252
0.333	0.04	0.04	0.00473	0.00363	0.0011	0.06543	0.06984	0.06533
0.333	0.04	0.04	0.00584	0.00451	0.00133	0.06991	0.07484	0.06954
0.333	0.04	0.04	0.00635	0.00492	0.00143	0.07189	0.07663	0.07096
0.333	0.04	0.04	0.00698	0.00543	0.00155	0.07405	0.07912	0.07281
0.333	0.04	0.04	0.00763	0.00596	0.00167	0.07615	0.08235	0.07438
0.333	0.04	0.07	0.0037	0.00353	0.00017	0.07305	0.07723	0.07568
0.333	0.04	0.07	0.0043	0.00404	0.00026	0.07598	0.0797	0.07857
0.333	0.04	0.07	0.00488	0.00455	0.00033	0.07852	0.08179	0.08142
0.333	0.04	0.07	0.00541	0.005	0.00041	0.08084	0.08434	0.08368
0.333	0.04	0.07	0.00605	0.00554	0.00051	0.08355	0.08761	0.08576
0.333	0.04	0.07	0.0066	0.006	0.0006	0.08575	0.08895	0.08807
0.333	0.04	0.07	0.00719	0.0065	0.00069	0.08813	0.09142	0.08956
0.333	0.04	0.07	0.00766	0.00689	0.00077	0.08938	0.09278	0.09169
0.333	0.04	0.07	0.00818	0.00734	0.00084	0.09288	0.0953	0.09354
0.333	0.05	0.00	0.003	0.00105	0.00195	0.06137	0.06524	0.04747
0.333	0.05	0.00	0.006	0.00319	0.00281	0.07628	0.0823	0.05763
0.333	0.05	0.00	0.009	0.00553	0.00347	0.07628	0.09517	0.0669
0.333	0.05	0.01	0.003	0.00101	0.00199	0.06149	0.06603	0.04729
0.333	0.05	0.01	0.006	0.00321	0.00279	0.0765	0.08259	0.05863
0.333	0.05	0.01	0.009	0.00554	0.00346	0.0765	0.09522	0.06772
0.333	0.05	0.01	0.003	0.00102	0.00198	0.06181	0.06646	0.04856
0.333	0.05	0.01	0.006	0.00322	0.00278	0.07666	0.0831	0.0598
0.333	0.05	0.01	0.009	0.00554	0.00346	0.07666	0.09579	0.06856

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.333	0.05	0.02	0.003	0.00112	0.00188	0.06247	0.06741	0.05298
0.333	0.05	0.02	0.006	0.00327	0.00273	0.07698	0.0832	0.06133
0.333	0.05	0.02	0.009	0.0056	0.0034	0.07698	0.09603	0.0697
0.333	0.05	0.02	0.003	0.00126	0.00174	0.06328	0.06847	0.05639
0.333	0.05	0.02	0.006	0.00341	0.00259	0.07801	0.08405	0.06549
0.333	0.05	0.02	0.009	0.00581	0.00319	0.07801	0.0969	0.07524
0.333	0.05	0.03	0.003	0.00157	0.00143	0.06582	0.07013	0.06334
0.333	0.05	0.03	0.006	0.00375	0.00225	0.08025	0.08618	0.07384
0.333	0.05	0.03	0.009	0.00606	0.00294	0.08025	0.09786	0.08118
0.333	0.05	0.04	0.003	0.0019	0.0011	0.06937	0.07388	0.06951
0.333	0.05	0.04	0.006	0.00419	0.00181	0.08339	0.08918	0.08106
0.333	0.05	0.04	0.009	0.00652	0.00248	0.08339	0.09973	0.08858
0.333	0.05	0.05	0.0029	0.00237	0.00053	0.06499	0.06854	0.0665
0.333	0.05	0.05	0.00368	0.00301	0.00067	0.06847	0.07222	0.06956
0.333	0.05	0.05	0.00436	0.00358	0.00079	0.07129	0.07516	0.07204
0.333	0.05	0.05	0.00498	0.00409	0.00089	0.0735	0.07719	0.07425
0.333	0.05	0.05	0.00549	0.00433	0.00116	0.07924	0.08319	0.07924
0.333	0.05	0.05	0.00594	0.0047	0.00124	0.081	0.08533	0.08138
0.333	0.05	0.05	0.00661	0.00523	0.00138	0.08366	0.08783	0.08341
0.333	0.05	0.05	0.00714	0.00563	0.00151	0.08581	0.09017	0.08499
0.333	0.05	0.05	0.0078	0.00617	0.00163	0.08846	0.0932	0.08757
0.333	0.05	0.07	0.00316	0.00286	0.0003	0.07736	0.0819	0.07897
0.333	0.05	0.07	0.00384	0.00343	0.00041	0.08073	0.085	0.08327
0.333	0.05	0.07	0.0044	0.00388	0.00052	0.08328	0.08802	0.08606
0.333	0.05	0.07	0.00505	0.00439	0.00066	0.08647	0.09024	0.08862
0.333	0.05	0.07	0.0056	0.00484	0.00076	0.08871	0.093	0.09079
0.333	0.05	0.07	0.00624	0.00537	0.00087	0.09128	0.0949	0.09295
0.333	0.05	0.07	0.00664	0.0057	0.00093	0.09321	0.09621	0.09378
0.333	0.05	0.07	0.00726	0.0062	0.00106	0.09547	0.09937	0.09646
0.333	0.05	0.07	0.00785	0.00669	0.00115	0.09781	0.10122	0.09896
0.333	0.06	0.06	0.00295	0.00236	0.00059	0.07846	0.08167	0.07918
0.333	0.06	0.06	0.00378	0.00301	0.00077	0.08294	0.08633	0.08345
0.333	0.06	0.06	0.00443	0.0035	0.00093	0.08603	0.08945	0.08622
0.333	0.06	0.06	0.00516	0.00409	0.00107	0.08912	0.09262	0.08923
0.333	0.06	0.06	0.00572	0.00453	0.00119	0.09155	0.09513	0.09135
0.333	0.06	0.06	0.00626	0.00496	0.0013	0.09387	0.09737	0.0939
0.333	0.06	0.06	0.00672	0.00532	0.0014	0.0958	0.0996	0.09508
0.333	0.06	0.06	0.00732	0.00579	0.00153	0.09807	0.10212	0.09734
0.333	0.06	0.06	0.00787	0.00622	0.00165	0.10105	0.10397	0.09921
0.333	0.06	0.06	0.00849	0.00673	0.00176	0.10265	0.10684	0.10076
0.333	0.06	0.07	0.00294	0.00224	0.0007	0.08642	0.08956	0.08806

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.333	0.06	0.07	0.00363	0.00286	0.00077	0.08951	0.09287	0.09087
0.333	0.06	0.07	0.0042	0.0033	0.0009	0.0924	0.09608	0.09359
0.333	0.06	0.07	0.00482	0.00378	0.00104	0.09538	0.09969	0.09634
0.333	0.06	0.07	0.00543	0.00427	0.00117	0.09821	0.10218	0.09925
0.333	0.06	0.07	0.00603	0.00473	0.00131	0.10072	0.10432	0.10154
0.333	0.06	0.07	0.00669	0.00526	0.00143	0.10346	0.10742	0.10391
0.333	0.06	0.07	0.00729	0.00573	0.00156	0.10584	0.10952	0.10563
0.333	0.06	0.07	0.00804	0.00632	0.00171	0.10883	0.11184	0.10879
0.333	0.07	0.03	0.00286	0.00093	0.00193	0.08106	0.08468	0.07405
0.333	0.07	0.03	0.00355	0.00119	0.00237	0.08493	0.08864	0.07735
0.333	0.07	0.03	0.0041	0.00147	0.00263	0.08807	0.09209	0.07961
0.333	0.07	0.03	0.00464	0.00186	0.00278	0.091	0.0948	0.08172
0.333	0.07	0.03	0.0052	0.00226	0.00294	0.09395	0.09806	0.08376
0.333	0.07	0.03	0.00588	0.00272	0.00316	0.09707	0.1009	0.08629
0.333	0.07	0.03	0.00641	0.00311	0.0033	0.09946	0.10348	0.08742
0.333	0.07	0.03	0.00704	0.00355	0.00349	0.10226	0.10672	0.08896
0.333	0.07	0.03	0.00759	0.00395	0.00365	0.10442	0.10865	0.09095
0.333	0.07	0.04	0.00287	0.00098	0.00189	0.08431	0.08694	0.08049
0.333	0.07	0.04	0.00344	0.00138	0.00207	0.08758	0.09147	0.08311
0.333	0.07	0.04	0.00406	0.0018	0.00226	0.09094	0.09514	0.08541
0.333	0.07	0.04	0.00469	0.00224	0.00245	0.09414	0.09802	0.08787
0.333	0.07	0.04	0.00529	0.00265	0.00263	0.09696	0.10106	0.09015
0.333	0.07	0.04	0.00583	0.00305	0.00279	0.09967	0.10304	0.0923
0.333	0.07	0.04	0.00643	0.00349	0.00294	0.1021	0.10603	0.09442
0.333	0.07	0.04	0.00703	0.00392	0.0031	0.10458	0.10916	0.09621
0.333	0.07	0.04	0.00762	0.00439	0.00323	0.1073	0.1121	0.0983
0.333	0.07	0.05	0.00283	0.0014	0.00144	0.08758	0.09133	0.08771
0.333	0.07	0.05	0.00343	0.00181	0.00162	0.09105	0.09471	0.09053
0.333	0.07	0.05	0.00409	0.00228	0.00182	0.09453	0.09811	0.09249
0.333	0.07	0.05	0.00467	0.0027	0.00197	0.09752	0.10174	0.09544
0.333	0.07	0.05	0.00521	0.00309	0.00213	0.1	0.10358	0.09763
0.333	0.07	0.05	0.0058	0.00354	0.00226	0.10265	0.1067	0.09967
0.333	0.07	0.05	0.0064	0.00397	0.00243	0.10519	0.10872	0.1017
0.333	0.07	0.05	0.00702	0.00443	0.00259	0.1079	0.11019	0.10345
0.333	0.07	0.05	0.00772	0.00496	0.00276	0.11071	0.11544	0.10581
0.333	0.07	0.06	0.00309	0.00195	0.00114	0.09236	0.09616	0.09339
0.333	0.07	0.06	0.00368	0.00238	0.00131	0.09561	0.09947	0.09631
0.333	0.07	0.06	0.00438	0.00289	0.00148	0.09895	0.10292	0.09936
0.333	0.07	0.06	0.00503	0.00338	0.00165	0.10215	0.10634	0.10195
0.333	0.07	0.06	0.00565	0.00386	0.00179	0.10497	0.10921	0.10385
0.333	0.07	0.06	0.00623	0.0043	0.00194	0.10743	0.11132	0.10683

R_b	C_x (m)	C_y (m)	Q_{xi} (m ³ /s)	Q_{xo} (m ³ /s)	Q_{yo} (m ³ /s)	h_{xi} (m)	h_{xo} (m)	h_{yo} (m)
0.333	0.07	0.06	0.00688	0.00479	0.00209	0.11016	0.11414	0.10848
0.333	0.07	0.06	0.00741	0.0052	0.00221	0.11239	0.11675	0.11035
0.333	0.07	0.06	0.00801	0.00567	0.00234	0.11452	0.11856	0.11219
0.333	0.07	0.07	0.00287	0.00226	0.00061	0.00576	0.09253	0.09048
0.333	0.07	0.07	0.0035	0.00274	0.00076	0.00559	0.09583	0.09365
0.333	0.07	0.07	0.00409	0.00319	0.0009	0.00545	0.09872	0.0961
0.333	0.07	0.07	0.00455	0.00355	0.001	0.00534	0.10114	0.09851
0.333	0.07	0.07	0.00521	0.00407	0.00114	0.00519	0.10374	0.10077
0.333	0.07	0.07	0.00602	0.00468	0.00134	0.00503	0.10745	0.10389
0.333	0.07	0.07	0.00664	0.00518	0.00146	0.0049	0.11026	0.10663
0.333	0.07	0.07	0.00711	0.00555	0.00156	0.00481	0.11219	0.10862
0.333	0.07	0.07	0.00795	0.00622	0.00173	0.00465	0.11587	0.11136
0.333	0.04	0.04	0.00264	0.002	0.00064	0.05572	0.05949	0.05665

