DESIGN AND ANALYSIS OF ONLINE CONVERSION ALGORITHMS FOR FINANCIAL TRADING

BY

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ABSTRACT

In an online trading problem, an online player (trader/investor) wants to trade an asset in hand with a desired asset with the goal of maximizing the profit by the end of a trading period. Online trading algorithms are classified as non-pre-emptive and pre-emptive algorithms. Non-pre-emptive algorithms (also known as Reservation Price Algorithms) restrict the trader to invest all or nothing at any time point. In contrast, pre-emptive algorithms allow the trader to trade any amount as she desires. The online currency conversion is a form of online trading where a trader looks for converting a currency in hand to another desired currency, e.g., (Dollars to Yens). The performance analysis of online trading and conversion algorithms is majorly done under the worst-case competitive analysis paradigm. Although a variety of algorithms are proposed in the literature for online conversion problems, the intrinsic problem settings and the inherent assumption of the underlying models are unrealistic and far from real-world scenarios. For instance, online algorithms for conversion problems assume a-priori knowledge about the lower and upper bounds of future price quotations. Although, this is a key requirement for the design of an optimal algorithm, under real-world settings such assumptions might not hold and can render the algorithms ineffective. Likewise, the algorithms also treat every input sequence as worst-case and consequently the decision making is based on a pessimistic approach. The pessimistic approach, in turn leads to lack of risk management for investors, which is a key component for real-world practical usability of such algorithms. Our research focus is the design of new online trading algorithms and models with the objective of profit maximization (or cost minimization), while considering the real-world settings. We propose an algorithm RP^* using novel update model with realistic price bounds. The key feature of the proposed model is to regularly update the price bounds based on inter day price fluctuation. The experimental competitive ratio $c^e(RP^*)$ is compared with the benchmark algorithm, where RP^* outperforms the benchmark algorithm for all 120 data subsets. Next, we consider the k-Min search problem of buying k units of assets with the objective of minimizing the total buying cost and propose a risk aware policy using risk and reward framework for better returns. The competitive ratio c_ω of the proposed k-Min-RR is bounded by $c_\omega \in [c^*/\omega, c^*\omega]$, i.e., c_ω improves if the forecast is true and c_ω never goes beyond $c^*\omega$ if the forecast is false. Further, we model the realistic worst-case sequences for a set of algorithms and analyse the performance gap between theoretical and practical worst-cases. The effectiveness of our proposed solutions is evaluated under theoretical analysis model to assure the worst-case performance bounds. To show the practical significance, we also perform the experimental simulations over the real-world stock exchange data. The maximum observed performance gap between c_{ec} and c_{wc} is 85.5%. The results of analytical and experimental measures highlight the success guarantee of our designed policies. The thesis work is concluded by open questions and new directions for future work.

خلاصة البحث

في مشكلة التجارة او التبادل عبر الإنترنت ، يريد المستثمر عبر الإنترنت بيع الأصول التي لديه مقابل الحصول على أصول يرغب بها من أجل زيادة الربح إلى أقصى حد بنهاية فترة التداول. لقد تم التحقيق في خوارزميات التداول عبر الإنترنت, و تم تقسيمها إلى قسمين و هي خوارزميات غير–استباقية و أستباقية. تقوم الخوارزميات الغير أستباقية و المعروفة أيضا بأسم باسم خوارزميات سعر الحجز) بتقييد المتداول بحيث أنه إما أن يستثمر كل شيء أو لا شيء في أي وقت. في المقابل ، تسمح الخوارزميات الأستباقية للمستثمر بالتداول بأي مبلغ يريده. تحويل العملات عبر الإنترنت هو شكل من أشكال التداول عبر الإنترنت حيث يبحث المستثمر عن تحويل عملة ما إلى عملة أخرى مرغوبة ، على سبيل المثال ، (الدولار إلى الين). يتم إجراء تحليل أداء التداول عبر الإنترنت وخوارزميات التحويل بشكل رئيسي في ظل نموذج التحليل التنافسي الأسوأ. على الرغم من اقتراح مجموعة متنوعة من الخوارزميات في الدراسات السابقة بمشاكل التحويل عبر الإنترنت ، إلا أن إعدادات المشكلة الجوهرية والافتراض المتأصل للنماذج الأساسية غير واقعية وبعيدة عن سيناريوهات العالم الحقيقي. على سبيل المثال ، تفترض الخوارزميات عبر الإنترنت الخاصة بمشكلات التحويل معرفة مسبقة بالحدود الدنيا والعليا لعروض الأسعار المستقبلية. على الرغم من أن هذا مطلب أساسي لتصميم خوارزمية مثالية ، إلا أنه في ظل إعدادات العالم الحقيقي ، قد لا تصمد هذه الافتراضات ويمكن أن تجعل الخوارزميات غير فعالة. وبالمثل ، تتعامل الخوارزميات أيضًا مع كل تسلسل إدخال على أنه أسوأ حالة ، وبالتالي فإن اتخاذ القرار يعتمد على النهج متشائم. يؤدي النهج المتشائم بدوره إلى الافتقار إلى إدارة المخاطر للمستثمرين ، وهو عنصر أساسي للاستخدام العملي في العالم الحقيقي لهذه الخوارزميات. ينصب تركيزنا البحثي على تصميم خوارزميات ونماذج جديدة للتداول عبر الإنترنت بهدف تعظيم الربح (تقليل التكلفة) ، مع دمج إعدادات العالم الحقيقي. نقترح خوارزمية باستخدام نموذج تحديث جديد بحدود أسعار واقعية وإلغاء التقلبات التعسفية. تم تصميم سياسة هجينة جديدة لتسهيل المشترين لشراء وحدات متعددة لبعض الأصول المرغوبة عندما يبدو عرض السعر جيدًا بما فيه الكفاية. ركزت هذه الرداسة أيضًا على التحقيق في هذه الخوارزمية باستخدام إطار عمل المخاطر والمكافآت لتحقيق عوائد أفضل. علاوة على ذلك ، نقوم بنمذجة المتواليات الواقعية (القريبة من مواقف العالم الحقيقي) لأسوأ حالة للخوارزميات المحددة ونحلل فجوة الأداء بين العالمين النظري والعملي. يتم تقييم فعالية حلول كسب المال المقترحة لدينا في إطار نموذج التحليل النظري لضمان حدود الأداء في أسوأ الحالات. لإظهار الأهمية العملية ، نقوم أيضًا بإجراء عمليات المحاكاة التجريبية على بيانات البورصة في العالم الحقيقي. تسلط نتائج التدابير التحليلية والتجريبية الضوء على ضمان نجاح سياساتنا المصممة لكسب المال. و طرحت هذه الأطروحة مجموعة من الأسئلة المفتوحة و أقترحت توجهات جديدة للعمل المستقبلي.

APPROVAL PAGE

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DECLARATION

I hereby declare that this thesis is the result of my own investigations, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at IIUM or other institutions.

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Signature .... Date ....30.6.2021.................

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To my beloved Mama Jan, Papa Jan

&

Hubby Jan

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LIST OF NOTATIONS

ON	An online algorithm
$\widehat{ON}$	An arbitrary online algorithm for buying problem
OPT	An optimum offline algorithm
OFF	An offline algorithm
$RAND$	The best randomized algorithm for a specified conversion problem
USD	Currency of United States, i.e., Dollar
DAX30	Deutscher Aktein Index: German stock market index for 30 major companies
S&P500	Standard & Poor's 500 Index: Market weighted index for 500 largest U.S. trading companies
CAC40	French stock market index of the top 40 stocks
FTSE100	Financial Times Stock Exchange 100: Market capitalization index of the top100 companies on the London Stock Exchange
IBEX35	Stock market index of the Bolsa de Madrid, Spain's principal stock exchange
Nikkei	Stock index of the Tokyo Stock Exchange Market
HSI	Hang Seng Index: Market capitalization weighted stock index of the Hong Kong Stock Exchange Market
RPP	Reservation Price Policy
TBP	Threat Based Policy
TBP′	Modified Threat Based Policy
BAL	Balanced Policy
DCA	Dollar Cost Averaging Policy
GBM	Geometric Brownian Motion
ED	Euclidean Distance
VS	V-Shift similarity
DTW	Dynamic time warping
BQ	Behavior quotient also called trend similarity or distance measure
U_p	Uni-directional preemptive algorithm (wealth is divisible)
U_{np}	Uni-directional non-preemptive (wealth is non-divisible)
B_p	Bi-directional Preemptive Algorithm (wealth is divisible)
B_{np}	Bi-directional Non-Preemptive (wealth is non-divisible)
$\mathbb{L}$	Length (number of days) of a complete trading horizon
p	Total number of investment periods (buy-sell phases) in complete trading horizon of duration $\mathbb{L}$
T	Length (number of days) of an investment period where $T = \mathbb{L}/p$
T'	Remaining number of days in the investment horizon; $T' = T - t + 1$
$\mathcal{I}$	Set of all input sequences
I	An input sequence $I \in \mathcal{I}$ with length T where $t = \{1,2,\dots,T\}$
Q	Worst-case time series
m	The predicted lower bound of the offered price sequence
M	The predicted upper bound of the offered price sequence
M'	True upper bound of the price sequence

m'	True lower bound of the price sequence
$\mathcal{M}'$	Forecasted upper bound with risk
$\mathcal{R}_c$	Ratio of the competitive ratio
$\mathcal{F}$	Set of input instances that belongs to a given forecast function
t	A given time point, e.g., day from interval T
t'	A time point for which a forecast is made
k	Number of units to buy/sell
z	Number of prices to be accepted for trading, where $i = \{1, \dots, z\}$ with $z \leq T$
ϕ	The predicted maximum fluctuation ratio between two price offers
k'	Threat duration where $k' \leq T$
q_t	Price quotation offered on some specific day $t \in T$
q_T	Price quotation offered on the last day T of the trading horizon
q^*	Reservation price (limit) to perform a buy or sell phase
q^{*^Y}	Reservation price (limit) when converting an asset D to some asset Y
q^{*^D}	Reservation price (limit) when converting an asset Y to some asset D
S_t	Amount of wealth/asset to be traded at a given time point t
Y_t	Accumulated amount of wealth/asset on a given time point t
D_t	Remaining amount of wealth/asset on a given time point t
T_x	Number of transactions taken by an online algorithm
ω	Risk tolerance level for an investor, where $\omega \in [1, r_{ON}]$
c	Competitive ratio
c^*	Optimal achievable competitive ratio
c_ω	Worst-case competitive ratio (using risk and reward)
$\tilde{c}$	Lower bound on competitive ratio for the best randomized algorithm for unit-directional non-preemptive conversion problem.
c_s	Smoothed competitive ratio
τ	Constant with $\tau > 0$
ϑ_1, ϑ_2	An upcoming price quotation q_{t+1} is bounded as $q_{t+1} \in [q_t\vartheta_1, q_t\vartheta_2]$, where $(0 < \vartheta_1 \leq \vartheta_2)$
α, β	An upcoming price quotation is bounded as $q_{t+1} \in \left[\frac{q_t}{\beta}, \alpha q_t\right]$, where $(\alpha > 1)$ and $(\beta > 1)$
$\hat{\alpha}$,$\hat{\beta}$	Two constants where $(0 < \hat{\alpha} < 1 < \hat{\beta} < \infty)$
P, R	Two given time series for finding similarity with similar size of n or varied size of n and m relative to P and R respectively
δ	Estimation Error in price bounds
Δ	Family of probability distributions
ϱ	A chosen probability distribution from family of distribution
r_{ON}	Risk value of an online algorithm ON
a_{ON}	Amortized cost of an online algorithm ON
R_{wo}	Relative worst order ratio
$g_t(q')$	Profit function with q' accepted price quotation
Buy_t	Number of units bought at time point t
Acc_t	Number of units accumulated after time point t
$R(\max)_{ON}$	Max/Max Ratio of a given online algorithm ON
$R(ro)_{ON}$	Random order measure of an online algorithm ON
$E[ON]$	Expected performance of an online algorithm ON

CHAPTER ONE

INTRODUCTION

For every problem there is a need of finding the solution and that solution is formally called Algorithm. This chapter presents the algorithms with its prime role in vast variety of computer science applications e.g., algorithm trading and financial stock exchanges. We present the online trading problem (buy/sell) and time series search (min/max) with problem settings. Currency conversion for uni-directional as well as bi-directional problem is discussed. We introduce the problem statement followed by the research questions and objectives. Then, thesis contributions are highlighted. After thesis organization section, the chapter concludes with summary.

1.1 ALGORITHMS AND COMPUTER SCIENCE

Algorithms are the core of computer science and form the basis for any problem solving or decision-making paradigm. One of the oldest documented algorithms (circa 300 BC) is the Euclid's algorithm for finding the greatest common divisor. The algorithms are designed based on different assumptions regarding the availability of input data, known probability distribution of the input data, and the decision-making power of the underlying policy (Schwarz, 2020). For the problems that are computationally expensive e.g., NP-hard problems with exponential computational complexity, the exact solutions are not possible (Vaziani, 2013). Thus, approximations algorithms are proposed that provide a fast (computationally efficient) solution at the expense of optimality. The solution of an approximation algorithm may not be an optimal but must not be arbitrarily bad as well (Vaziani, 2013). Generally, NP-hard problems are offline, i.e., the whole input sequence is available to the algorithm before the processing begins.

In some situations, the input data might not be available to the algorithm at the start, and such problems are called online problems. The solutions for the online problems are required in diversified real-world situations, e.g., online financial games where stock market data is not available beforehand, instead it is revealed as a single data item at a time in a serial fashion (Mohr et al., 2014, Yang et al., 2020). Besides financial games, online algorithms are used in various other real-world problems such as the best secretary selection (Ferguson, 1989; Babaioff et al., 2008), market clearing (Blum et al., 2006), graphs (Barriers et al., 2018), self-organizing data structures (Albers and Janke, 2020), scheduling and load balancing (I'm et al., 2011; Khalil et al., 2020, Ahmad et al., 2020), bin-packing (Balogh et al., 2020), and finance (Fung, 2019; Schwarz, 2020).

A financial market, (such as forex-Foreign Currency Exchange) is a place where buyers and sellers trade for financial securities and assets, e.g., gold, stocks and bonds. The advancement of technology has led the traders towards electronic financial markets for automated trading. Algorithmic trading (Blum et al., 2006; Chang and Johnson, 2008) also known as automated trading is the utilization of trading algorithms and computer systems (software) to work without human intervention for generating revenues in global/local financial markets (McKenzie, 2019). Computer systems are exceptional traders provided with the right financial money-making algorithms and strategies.

The rise of algorithmic trading provides a number of opportunities for the layman to invest in the financial markets. Systems based on algorithmic trading offer multiple advantages over human trading. Computers based trading systems stay active

$24/7$ and quickly respond to the market trends and variations. These systems can perform precise data analysis and quick decision-making. Traders are in search of profit-making strategies and techniques while handing over the actual part of trading and investment to the automated applications and software.

$NASDAQ$ is the computerized trading system of the United States using proprietary algorithms? The financial transactions of $NASDAQ$ are presented as auctions, instead of the traditional communicational order flow. It is estimated that 50% of the stock trading volume in the United States is currently from high-frequency algorithmic trading (Nasdaq, 2018). As of 2018, the algorithmic trading market was worth USD 10346.6 million in year 2018. It is expected to attain Compound Annual Growth Rate (CAGR) of 10.7% over the forecast period (2019-2027), i.e., surpass 25,257 million by year 2027[1].

1.2 ONLINE FINANCIAL GAMES

An online financial game consists of two players, an online algorithm (ON) and an optimum offline algorithm called adversary (OPT). As the game starts, a sequence of price quotations is revealed (each price per unit time). The online algorithm selects one price quotation a, the adversary also selects some price quotation b. If $a > b$, the online algorithm wins otherwise the adversary wins. However, if the adversary has complete knowledge of all price quotations before the game starts then b must be the highest price quotation in the whole input price sequence. In this case, the online algorithm loses, and the performance ratio of online algorithm is $\frac{b}{a}$. The online algorithm wins, if and only if $a = b$ which gives a performance ratio of 1. Technically, the performance ratio is actually competitive ratio which is defined later in Section 2.3.2. The game ends, when a price quotation is chosen. The game may continue, if the algorithm needs multiple selection of price quotations.

[1]Visit https://www.coherentmarketinsights.com/ for global algorithmic trading, e.g., Equities, Forex, Funds and Commodities

In any financial trading and conversion problem, the goal of a trader is to select the best possible price quotation(s) to maximize (minimize) the profit (cost) by the end of trading horizon. The problem is a specialized form of the optimal stopping theory (Dynkin, 1963; Kleinberg, 2005; Babaioff et al., 2008). A direct application of this theory is the online secretary selection problem (Ferguson, 1989) which deals with the selection of a single employee among T available candidates.

The candidates are interviewed in a random sequence. Because of the online nature of problem, every candidate must be accepted or rejected right after the interview. An applicant is only accepted when he/she performs better than all the previously conducted interviews. The decision is solely based on the relative ranks of all the interviewed candidates so far. Once a candidate is selected for the position to fill, the game ends. If the last candidate T is reached while rejecting all $T-1$ candidates, he/she must be accepted for the position. Moreover, a rejected candidate cannot be re-considered during the selection procedure. An optimal stopping algorithm rejects the initial T/e appeared candidates and then chose the one who is better than all the rejected candidates. When $T \to \infty$, the probability of choosing the best candidate from the available list of candidates is about 37%, i.e., $1/e \approx 0.37$ (Dynkin, 1963). The secretary problem using multiple choice algorithm with its applicability for online auctions is investigated by Kleinberg (2005). Babaioff et al. (2008) studied the generalized secretary problem under uniform random order of candidates. A generalization of the classical secretary problem called Matroid secretary problem is introduced by Babaioff et al. (2007) and further improved by Chakraborty and Lachish (2012).

In this work, we focus on online financial algorithms for trading and conversion problems. In an online financial algorithm, the online player is unaware of upcoming input sequence and selects some favourable price quotation(s) without any knowledge of future price quotations. The adversary is malicious, i.e., she knows the input sequence before the game starts. She is also aware of each decision taken by the online algorithm. Thus, the goal of any online algorithm is to minimize the competitive ratio as small as possible by reducing the gap between the performance of an online player and the adversary, i.e., ON vs. OPT. The popular online financial games include time series min-search and max-search with various practical applications (El-Yaniv et al., 2001; Hu et al., 2005; Iqbal and Ahmad, 2015; Fung, 2019).

1.3 ONLINE TRADING AND PROBLEM SETTINGS

1.3.1 Trading Problem and Time Series Search

Financial markets such as stock exchanges are highly volatile (Verbing, 2014). The prime goal of an investor in any financial market is the profitability of the underlying trading policy. In an online trading problem, an online player searches for the minimum possible price quotation to buy some desired asset with the goal of minimizing the overall cost by the end of trading horizon (Mohr et al., 2014, Fung 2021). Similarly, an online player may look for the maximum possible price quotation to sell an asset in hand with the goal of maximizing the overall returns at the end of trading period.

The problem of min-search/max-search is known as unit-directional conversion which involves either a buy or a sell phase (Damaschke et al., 2009; Zhang et al., 2011; Zhang et al., 2012; Xu et al., 2011; Iqbal and Ahmad, 2015). When the search for favorable price quotations can be performed both ways (a buy phase followed by a sell phase or vice versa), this is called bi-directional conversion (Kao and Tate, 1999; Mohr and Schmidt, 2009; Schmidt et al., 2010; Iqbal et al., 2012; Schroeder et al., 2016; Schmidt, 2017; Fung, 2019; Schwarz, 2020). In bi-directional conversion multiple min and max searches (buy and sell steps) are performed. A profitable bi-directional trading policy is one, where an investor can maximize the difference between selling and buying prices. Investor must be capable to buy a desired asset at some minimum possible price and sell it at a maximum possible price. In the real-world, future prices in any stock market are unknown. This fact limits the investor's ability to decide whether an observed price offer is good for trading or not.

1.3.2 k-Min Search Problem

k-Min is a special case of online uni-directional conversion problems (Iqbal & Ahmad, 2015; Lorenz et al., 2009). k-Min search problem deals with the scenario where a trader (online player) is allowed to buy k units of a desired asset with goal of minimizing the total buying cost. At each time point t during an investment period, where $t = \{1,2,\ldots,T\}$, the online player is offered a price q_t. The player has to either accept the price offer to buy some units of desired asset or reject the price with the hope of getting a better price offer later (Iqbal and Ahmad, 2015). The player has no knowledge of any future price quotations. The game ends when the player has bought k units of asset or the last price offer q_T is unfolded. If there are still any units left to buy, the player is forced to buy at the last price offer q_T. The length of investment horizon T may or may not be known to the player before the game starts. If T is unknown to the player, she is informed right before revealing the last price offer. In any case, T is assumed to be finite and fixed.

1.3.3 Problem Settings

In the classical online conversion problem, the trader (online player) looks for a favorable price to convert some initial wealth/currency in hand D (say Dollars) into a desired currency or asset Y (say Yens) with the goal of maximizing the amount of Y by the end of trading horizon. The problem can be studied as unit-directional conversion (conversion from D to Y) and bi-directional conversion (conversion from D to Y and then Y back to D). Assume that the trade starts with one-unit dollar in hand. Trader gets a good price offer for conversion and trades one dollar for 1000 yens during the trading horizon (see Figure 1.1).

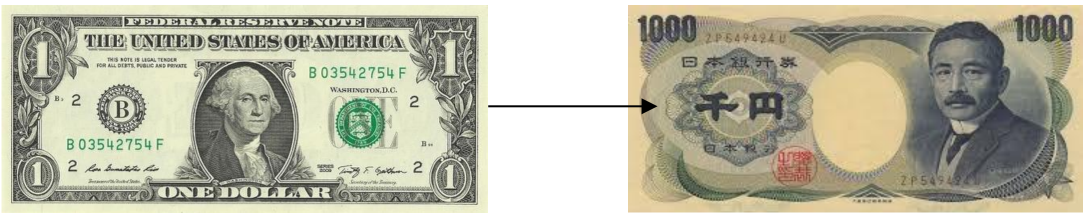

Figure 1.1 Uni-directional Conversion from Dollars to Yens

The trade may be continued when trader starts looking for a favorable price offer for converting 1000 yens back to dollars. Trader converts all yens back to dollar with final returns of 100 dollars by the end of trading horizon. In the real-world, overall returns of financial investors rely on the profitability of underlying trading algorithms and policies.